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A DIGITAL COMPUTER SOLUTION OF THE FALKNER AND SKAN BOUNDARY LAYER EQUATION

JAMES A. INGLIS

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A DIGITAL COMPUTER SOLUTION OF THE FALKNER AND SKAN BOUNDARY LAYER EQUATION

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James A. Inglis

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A DIGITAL COMPUTER SOLUTION
OF THE
FALKNER AND SKAN BOUNDARY LAYER EQUATION

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by

James A. Inglis

Lieutenant, Royal Canadian Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

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A DIGITAL COMPUTER SOLUTION
OF THE
FALKNER AND SKAN BOUNDARY LAYER EQUATION

by

James A. Inglis

This work is accepted as fulfilling
the thesis requirements for the degree of
MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING
from the
United States Naval Postgraduate School

Abstract.

The Falkner and Skan boundary layer equation is presented and the properties of the solutions being sought are discussed. The problem is programmed for solution with a digital computer by two methods, namely, by the Milne Five-Point Method and the Runge-Kutta Method. Several solutions are listed and the two methods of solution are compared. The results obtained are compared with, and found to confirm, the solutions produced by D.R. Hartree.

Acknowledgement.

The writer wishes to express his appreciation for the encouragement and helpful advice given him by Professor Gilles Cantin of the Department of Mechanical Engineering.

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Notation.*

a - a constant of proportionality

E_1, E_2, E_3 - constants denoting convergence requirements

f - a function of the boundary layer coordinate, η

f', f'', f''' - derivatives of f with respect to η

h - step-width in the η direction

L - a characteristic length of the immersed body

m - a constant, related to β

p - pressure

Re - a characteristic Reynolds number, $= \frac{U_0 L}{\nu}$

t - time

u, v - velocity components in the directions parallel to and normal to the boundary

u, v, w - velocity components in rectangular coordinates

u_1 - velocity at the outer limit of the boundary layer

U_0 - free-stream velocity

x, y - length coordinates measured tangential to and normal to the boundary

x, y, z - rectangular coordinates

X, Y, Z - body forces per unit mass

---

* The notation listed here is not applicable to Appendices B and C.

β - a constant, $= \frac{2m}{m+1}$

δ, δ_1 - boundary layer thickness

η - non-dimensional boundary layer coordinate normal to the boundary

ν - kinematic viscosity

ψ - non-dimensional stream function

ρ - density

1. Introduction.

The Falkner and Skan boundary layer equation,

$$f''' + f f'' + \beta(1 - f'^2) = 0 , \tag{1}$$

where f is a function of η , with the boundary conditions,

$$f = 0, f' = 0, \text{ at } \eta = 0, \text{ and } f' \to 1 \text{ as } \eta \to \infty , \tag{2}$$

describes fluid motion in the laminar boundary layer formed as a result of steady, incompressible, two-dimensional flow past an immersed body, such as a sharp-edged wedge, a circular cylinder, or a parabolic cylinder. This equation was first derived, in a slightly different form, by V.M. Falkner and Miss S.W. Skan [1][1]. Equation (1) was later solved numerically by D.R. Hartree [2] for several values of the parameter β.

This report describes two methods by which equation (1) can be solved with a digital computer. Solutions have been produced with a CDC 1604 Digital Computer by both methods illustrated and a comparison of the two methods is given. Solutions for several values of β, giving $f(\eta)$, $f'(\eta)$, $f''(\eta)$, and $f'''(\eta)$, are listed in Appendix D.

---

[1] Numbers in square brackets refer to the Bibliography.

2. The Falkner and Skan Equation.

Prandtl's boundary layer equation for steady, incompressible flow [3] is, in dimensionless form:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{dp}{dx} + \frac{1}{Re} \frac{\partial^2 u}{\partial y^2} . \tag{3}$$

Falkner and Skan assumed that

$$u_1 = a x^m , \tag{4}$$

where u_1 is the fluid velocity at the outer limit of the boundary layer, x is the distance, measured tangential to the boundary, from the forward stagnation point, and a is a constant of proportionality. It has been found [4],[5] that equation (4) closely describes the flow over the forward portion of a sharp-edged wedge of included angle $\pi\beta$, where the relationship between β and m is given by

$$\beta = \frac{2m}{m+1} , \tag{5}$$

and over circular and parabolic cylinders.

Next define a non-dimensional boundary layer coordinate,

$$\eta = y \sqrt{\frac{m+1}{2} \cdot \frac{u_1 Re}{x}} , \tag{6}$$

and a stream function,

$$\psi = \sqrt{\frac{2}{m+1} \cdot \frac{u_1 x}{Re}} \, f , \tag{7}$$

where f = f(η), a function of η only. The stream function ψ satisfies the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 , \tag{8}$$

namely,

$$u = \frac{\partial \psi}{\partial y} , \quad v = - \frac{\partial \psi}{\partial x} . \tag{9}$$

Now the introduction of equations (4) through (9) into equation (3) yields, after simplification, the Falkner and Skan equation (1).

For a complete derivation of equations (3) and (1) see Appendix A.

Those portions of the solution of equation (1) which are of particular interest are the variation of $f'(\eta)$ with η and the value of $f''(0)$, that is, the value of the unknown boundary condition at $\eta = 0$. It can be shown that

$$f' = \frac{u}{u_1} \; ; \tag{10}$$

that is, the variation of $f'(\eta)$ gives the velocity profile through the boundary layer.

The value of $f''(0)$ is necessary for the determination of the local coefficient of friction at the boundary, since the frictional shearing stress at the boundary is proportional to $\left.\frac{\partial u}{\partial y}\right)_{\eta=0}$ and

$$\left.\frac{\partial u}{\partial y}\right)_{\eta=0} = u_1\sqrt{\frac{m+1}{2}\cdot\frac{u_1 Re}{x}}\; f''(0) \; . \tag{11}$$

A particular solution of equation (1) which is of special interest is the case of boundary layer separation for which it is known that $f''(0) = 0$. In this case, the value of β becomes the unknown condition and the particular value of β which satisfies the equation is an essential part of the solution.

Under certain conditions, equation (1) is applicable to compressible flow [6],[7]. In this case, the solution $f(\eta)$ is required for the solution of the energy equation which is essential in the determination of heat transfer coefficients. The case of compressible flow is not considered in this report.

3. The Boundary Conditions.

In addition to the boundary conditions (2) given with the differential equation (1), certain additional requirements were imposed upon the solutions illustrated in this report. In Figures 1, 2, and 3, curves of f', f'', and f''' versus η have been plotted for various values of the parameter β. From an examination of these curves, the following conditions will be noted.

1. The range of f' is $0 \leqslant f' \leqslant 1$; that is, f' is always positive and approaches the asymptote $f' = 1$ from below.

2. $f'' \geqslant 0$ and $f'' \rightarrow 0$ as $\eta \rightarrow \infty$; that is, f'' is always positive and approaches the asymptote $f'' = 0$ from above.

3. $f''' \rightarrow 0$ from below as $\eta \rightarrow \infty$; even though f''' may be initially positive (as it is for $\beta < 0$), it must become and remain negative and approach the asymptote $f''' = 0$ from below.

In programming the solution of the Falkner and Skan equation (1), the conditions noted above were included with the boundary conditions which were imposed upon the solution being produced. Failure to satisfy any one of these specified conditions caused rejection of the solution being generated.

4. The Methods of Solution.

Two well-known numerical methods were chosen for the solution of equation (1). They are the "Milne Five-Point Method" [8],[9] and the "Runge-Kutta Method" [10]. The primary reason for choosing two methods to solve this problem, when one only is sufficient, is that the absolute accuracy of a solution produced by either method is not known and a detailed error analysis is difficult to perform. It is felt that if good agreement can be obtained between solutions by the two methods, then the solutions can reasonably be accepted as being accurate.

In any numerical integration scheme, the reliability of the results is very much dependent upon the selection of a suitable step-width,

$$h = \eta_{n+1} - \eta_n . \tag{12}$$

In general, the smaller the step-width, h, the more reliable will be the results. However, it must be remembered that one pays a penalty in that the number of points which must be evaluated increases as h decreases. Even with a high-speed digital computer, this factor must be borne in mind. The optimum value of the step-width was taken to be the maximum value of h which would give results which were reproducable to the same (desired) number of significant figures when a smaller value of h was used. The solutions listed in this report were calculated to eight places of decimals. In the case of the Milne Method, the optimum value was $h = 0.01$, and for the Runge-Kutta Method, $h = 0.001$ was required. This meant that the Milne Method

produced a solution in approximately one-tenth of the time required by the Runge-Kutta Method.

A detailed description of both methods is given in Appendices B and C. Both the Milne Method and the Runge-Kutta Method were programmed in FORTRAN language [11],[12] and were used with the CDC 1604 Digital Computer. The complete programs are described and listed also in Appendices B and C.

5. Convergence Criteria.

Inasmuch as the solution of the Falkner and Skan equation (1) approaches an asymptote as $\eta \rightarrow \infty$, the question of convergence criteria naturally arises; that is, what is the cut-off point of the solution and what requirements must be fulfilled at the cut-off point? Furthermore, in order to preclude the possibility of the computer computing indefinitely on any trial solution in the event that the trial solution happened to approach a horizontal asymptote, $f' < 1$, an upper limit of the variable η must be arbitrarily fixed.

Figures 1, 2, and 3 show that the various solutions rapidly approach their asymptotic value. In the preliminary stage, using Hartree's work as a guide, an upper limit of $\eta = 10.$ was chosen and this proved to be satisfactory for all solutions produced and did not require revision.

The convergence requirements which were imposed at the cut-off point are as follows:

1. $(1 - f') \leqslant E_1$ (13)
2. $f'' \leqslant E_2$ (14)
3. $- f''' \leqslant E_3$ (15)

The value of E_1 was arbitrarily chosen to be $E_1 = 1.0 \times 10^{-8}$. If the exact solution to equation (1) were known, the values of E_2 and E_3 corresponding to a given value of E_1 could be calculated, but, since such a solution is not known, the values of E_2 and E_3 must also be set arbitrarily. A number of preliminary runs were made in which the convergence requirements on f'' and f''' were omitted. From the

results so obtained, it was noted that for $E_1 \sim 10^{-8}$, f" was approximately one order of magnitude greater than (1 - f') and - f''' was approximately one order of magnitude greater than f". Based upon this observation, it was arbitrarily decided to set $E_2 = 10. \times E_1 = 1.0 \times 10^{-7}$, and $E_3 = 10. \times E_2 = 1.0 \times 10^{-6}$. These are the convergence criteria which were imposed upon the solutions which were produced by the methods of this report.

6. Results.

Several solutions which have been produced by the Milne Method are tabulated in Appendix D. These solutions are quoted to eight places of decimals. A number of the cases given were also solved by the Runge-Kutta Method. As mentioned earlier, the values of the step-width, h, that were used are 0.01 for the Milne Method and 0.001 for the Runge-Kutta Method. In each case compared, the solutions produced by the two methods were identical to seven places of decimals, and any discrepancy in the eighth-place was usually small.

Table I on the following page indicates the nature of the differences which exist between solutions produced by the two methods.

Hartree [2] gives solutions of $f'(\eta)$ for several values of β to four places of decimals. If the corresponding solutions listed in Appendix D are rounded off to four places, they are found to agree with Hartree's solutions to within +.0005 to -.0003, with the exception of two entries, which are likely to be typographical errors; these exceptions are as follows:

β	η	f' (Hartree)	f' (Appendix D)
0.5	1.6	0.8860	0.8760_{98}
2.0	0.8	0.7858	0.7958_{7} .

These differences of +.0005 to -.0003 are within Hartree's estimated limits of error.

The Milne Five-Point Method, which makes use of "predictor" and "corrector" formulae in sequence, is said to be exact for a seventh degree polynomial [9]; for any other regularly behaved function, the

Table I. Maximum Differences Between Runge-Kutta Method Solutions and Milne Method Solutions.

Specified Parameter		Differences			
$f''(0)$	β_0	$f(\eta)$	$f'(\eta)$	$f''(\eta)$	$f'''(\eta)$
0.0	0. x 10^{-8}	2. x 10^{-8}	1. x 10^{-8}	1. x 10^{-8}	1. x 10^{-8}
β	$f''(0)$				
-.19	0. x 10^{-8}	1. x 10^{-8}	0. x 10^{-8}	1. x 10^{-8}	2. x 10^{-8}
-.18	0.	2.	1.	1.	2.
-.16	0.	1.	0.	1.	1.
-.14	0.	2.	1.	1.	3.
-.10	0.	2.	1.	1.	3.
0.0	0.	2.	1.	0.	0.
0.1	0.	1.	0.	0.	1.
0.2	0.	2.	1.	1.	2.
0.3	0.	1.	1.	1.	1.
0.4	0.	1.	1.	1.	4.
0.5	0.	1.	1.	1.	2.
0.6	0.	1.	1.	1.	3.
0.7	0. x 10^{-9}	1.	1.	0.	2.
0.8	0.	2.	1.	1.	3.
0.9	0.	1.	0.	1.	5.
1.0	0.	1.	1.	1.	4.
1.2	0.	1.	1.	1.	3.
1.6	2.x 10^{-10}	1.	1.	1.	7.
2.0	0.	1.	1.	2.	10.
2.4	0.	1.	1.	1.	7.

exact solution lies between the "predicted" value and the "corrected" value. Furthermore, for a seventh degree polynomial, seventh order differences should vanish, and, for other functions, seventh order differences should be small and free from erratic changes in magnitude.

In order to note the behaviour of these items, the Milne Method solution was programmed to give, in addition to the final solution, the predicted value of $f''(\eta)$, the difference between the second estimate of $f''(\eta)$ and its predicted value, and a table of differences to the seventh order for the final solution of $f'(\eta)$. On the following four pages are shown this information as obtained for the solutions of $\beta = -.195$ and $\beta = 0.7$. The column headings shown have the following meanings:

F = f ,
DF = f' ,
D2F = f" ,
D3F = f''' ,
PRED D2F = Predicted value of f" ,
O3 = (Second estimate of f") - (Predicted value of f") ,
D7 = Seventh order difference .

It was found that O3 was, in every case, less than $1. \times 10^{-8}$. Values of D7 were found to be less than $1. \times 10^{-6}$ and its behaviour was not considered to be erratic. From these indications, it appears safe to say that the Milne Method solutions are accurate to at least six places of decimals. Furthermore, the close agreement between the

Table II. Milne Method Sample Solution No. 1.

```
FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    = -.1950000000

D2F(0) =   .0551718000

 ETA          F             DF             D2F            D3F          PRED D2F       C3

  .000     .00000000     .00000000     .05517180      .19500000     .05517180     .0E+00
  .100     .00030836     .00649216     .07467085      .19496876     .07467085    -.2E-11
  .200     .00136342     .01493391     .09416193      .19482813     .09416193     .0E+00
  .300     .00336008     .02532379     .11363002      .19449314     .11363002     .0E+00
  .400     .00649300     .03765835     .13305062      .19385956     .13305062     .0E+00
  .500     .01095636     .05193115     .15238790      .19280450     .15238790     .4E-11
  .600     .01694349     .06813153     .17159281      .19118745     .17159281     .0E+00
  .700     .02464639     .08624319     .19060145      .18885198     .19060145     .0E+00
  .800     .03425506     .10624262     .20933360      .18562820     .20933360     .4E-11
  .900     .04595677     .12809745     .22769150      .18133629     .22769150    -.4E-11
 1.000     .05993498     .15176460     .24555908      .17579109     .24555908     .4E-11
 1.100     .07636826     .17718846     .26280178      .16880811     .26280178     .7E-11
 1.200     .09542891     .20429905     .27926688      .16021093     .27926688    -.7E-11
 1.300     .11728145     .23301028     .29478481      .14983992     .29478481     .0E+00
 1.400     .14208088     .26321831     .30917127      .13756232     .30917127     .0E+00
 1.500     .16997093     .29480030     .32223050      .12328327     .32223050     .7E-11
 1.600     .20108200     .32761341     .33375963      .10695749     .33375963    -.1E-10
 1.700     .23552923     .36149440     .34355422      .08860069     .34355422     .1E-10
 1.800     .27341039     .39625978     .35141489      .06830026     .35141489    -.7E-11
 1.900     .31480392     .43170667     .35715498      .04622393     .35715498     .1E-10
 2.000     .35976710     .46761453     .36060884      .02262545     .36060884    -.1E-10
 2.100     .40833435     .50374766     .36164062     -.00215382     .36164062     .2E-10
 2.200     .46051591     .53985862     .36015282     -.02768833     .36015282    -.2E-10
 2.300     .51629684     .57569248     .35609439     -.05347767     .35609439     .3E-10
 2.400     .57563658     .61099181     .34946742     -.07896187     .34946742    -.3E-10
 2.500     .63846890     .64550228     .34033211     -.10354274     .34033211     .3E-10
 2.600     .70470255     .67897859     .32880934     -.12661010     .32880934    -.3E-10
 2.700     .77422245     .71119055     .31508034     -.14757171     .31508034     .3E-10
 2.800     .84689151     .74192900     .29938337     -.16588467     .29938337    -.4E-10
 2.900     .92255301     .77101120     .28200724     -.18108599     .28200724     .4E-10
 3.000    1.00103345     .79828544     .26328191     -.19281963     .26328191    -.4E-10
 3.100    1.08214588     .82363457     .24356667     -.20085758     .24356667     .4E-10
 3.200    1.16569347     .84697828     .22323664     -.20511307     .22323664    -.4E-10
 3.300    1.25147323     .86827397     .20266835     -.20564445     .20266835     .4E-10
 3.400    1.33927978     .88751616     .18222543     -.20264940     .18222543    -.4E-10
 3.500    1.42890897     .90473453     .16224539     -.19645008     .16224539     .4E-10
 3.600    1.52016125     .91999072     .14302817     -.18747055     .14302817    -.4E-10
 3.700    1.61284466     .93337410     .12482740     -.17620872     .12482740     .5E-10
 3.800    1.70677736     .94499685     .10784450     -.16320526     .10784450    -.5E-10
 3.900    1.80178965     .95498854     .09222598     -.14901242     .09222598     .5E-10
 4.000    1.89772541     .96349065     .07806379     -.13416491     .07806379    -.5E-10
 4.100    1.99444306     .97065124     .06539833     -.11915520     .06539833     .6E-10
 4.200    2.09181593     .97662005     .05422370     -.10441441     .05422370    -.5E-10
 4.300    2.18973225     .98154420     .04449449     -.09029968     .04449449     .6E-10
 4.400    2.28809466     .98556459     .03613358     -.07708787     .03613358    -.6E-10
 4.500    2.38681946     .98881318     .02904019     -.06497524     .02904019     .7E-10
 4.600    2.48583561     .99141100     .02309781     -.05408203     .02309781    -.6E-10
 4.700    2.58508361     .99346694     .01818133     -.04446069     .01818133     .7E-10
 4.800    2.68451416     .99507722     .01416338     -.03610662     .01416338    -.6E-10
 4.900    2.78408699     .99632541     .01091938     -.02897006     .01091938     .8E-10
 5.000    2.88376957     .99728295     .00833152     -.02296799     .00833152    -.7E-10
 5.100    2.98353591     .99800996     .00629146     -.01799544     .00629146     .9E-10
 5.200    3.08336555     .99855625     .00470202     -.01393538     .00470202    -.7E-10
 5.300    3.18324250     .99896253     .00347800     -.01066690     .00347800     .1E-09
 5.400    3.28315449     .99926158     .00254620     -.00807169     .00254620    -.7E-10
 5.500    3.38309212     .99947944     .00184493     -.00603861     .00184493     .1E-09
 5.600    3.48304835     .99963654     .00132312     -.00446676     .00132312    -.8E-10
 5.700    3.58301793     .99974865     .00093919     -.00326711     .00093919     .1E-09
 5.800    3.68299699     .99982785     .00065985     -.00236311     .00065985    -.8E-10
 5.900    3.78298271     .99988323     .00045887     -.00169036     .00045887     .1E-09
 6.000    3.88297307     .99992155     .00031585     -.00119585     .00031585    -.8E-10
 6.100    3.98296662     .99994781     .00021520     -.00083676     .00021520     .2E-09



 6.200    4.08296235     .99996561     .00014513     -.00057913     .00014513    -.8E-10
 6.300    4.18295955     .99997756     .00009688     -.00039648     .00009688     .2E-09
 6.400    4.28295773     .99998550     .00006401     -.00026851     .00006401    -.7E-10
 6.500    4.38295656     .99999072     .00004187     -.00017989     .00004187     .2E-09
 6.600    4.48295581     .99999412     .00002711     -.00011922     .00002711    -.6E-10
 6.700    4.58295534     .99999630     .00001737     -.00007817     .00001737     .3E-09
 6.800    4.68295505     .99999770     .00001102     -.00005071     .00001102    -.4E-10
 6.900    4.78295487     .99999858     .00000692     -.00003255     .00000692     .4E-09
 7.000    4.88295475     .99999914     .00000430     -.00002067     .00000430    -.2E-11
 7.100    4.98295469     .99999948     .00000265     -.00001299     .00000265     .5E-09
 7.200    5.08295464     .99999969     .00000161     -.00000807     .00000161     .6E-10
 7.300    5.18295462     .99999981     .00000097     -.00000497     .00000097     .7E-09
 7.400    5.28295460     .99999989     .00000058     -.00000302     .00000058     .2E-09
 7.500    5.38295460     .99999993     .00000034     -.00000182     .00000034     .9E-09
 7.600    5.48295459     .99999996     .00000020     -.00000108     .00000020     .4E-09
 7.700    5.58295459     .99999998     .00000012     -.00000064     .00000011     .1E-08
 7.800    5.68295459     .99999998     .00000007     -.00000037     .00000007     .6E-09
 7.900    5.78295458     .99999999     .00000004     -.00000021     .00000004     .2E-08
 7.920    5.80295458     .99999999     .00000003     -.00000019     .00000003     .9E-09
```

Table III. Table of Differences for Sample Solution No. 1.

```
TABLE OF DIFFERENCES FOR

BETA   = -.1950000000

D2F(0) =  .0551718000

ETA        DF          D1          D2          D3          D4          D5          D6          D7

 .0   .00000000
 .1   .00649216   .00649216
 .2   .01493391   .00844175   .00194960
 .3   .02532379   .01038987   .00194812  -.00000148
 .4   .03765835   .01233456   .00194468  -.00000344  -.00000196
 .5   .05193115   .01427280   .00193825  -.00000644  -.00000300  -.00000104
 .6   .06813153   .01620038   .00192758  -.00001067  -.00000423  -.00000123  -.00000017
 .7   .08624319   .01811166   .00191128  -.00001630  -.00000563  -.00000140  -.00000018   .00000001
 .8   .10624262   .01999944   .00188778  -.00002350  -.00000720  -.00000156  -.00000016   .00000002
 .9   .12809745   .02185483   .00185539  -.00003239  -.00000889  -.00000170  -.00000013   .00000003
1.0   .15176460   .02366715   .00181232  -.00004307  -.00001069  -.00000179  -.00000010   .00000003
1.1   .17718846   .02542386   .00175671  -.00005561  -.00001253  -.00000185  -.00000005   .00000005
1.2   .20429905   .02711060   .00168674  -.00006998  -.00001437  -.00000184   .00000001   .00000006
1.3   .23301028   .02871123   .00160063  -.00008611  -.00001613  -.00000176   .00000008   .00000007
1.4   .26321831   .03020803   .00149681  -.00010382  -.00001772  -.00000159   .00000017   .00000009
1.5   .29480030   .03158199   .00137395  -.00012286  -.00001903  -.00000132   .00000027   .00000010
1.6   .32761341   .03281311   .00123112  -.00014283  -.00001997  -.00000094   .00000038   .00000011
1.7   .36149440   .03388099   .00106788  -.00016325  -.00002042  -.00000044   .00000050   .00000012
1.8   .39625978   .03476537   .00088438  -.00018350  -.00002025   .00000017   .00000061   .00000011
1.9   .43170667   .03544689   .00068152  -.00020286  -.00001937   .00000088   .00000072   .00000011
2.0   .46761453   .03590786   .00046097  -.00022055  -.00001769   .00000168   .00000080   .00000008
2.1   .50374766   .03613313   .00022527  -.00023570  -.00001515   .00000254   .00000086   .00000006
2.2   .53985862   .03611096  -.00002217  -.00024744  -.00001174   .00000341   .00000087   .00000001
2.3   .57569248   .03583386  -.00027710  -.00025493  -.00000749   .00000425   .00000084  -.00000003
2.4   .61099181   .03529934  -.00053452  -.00025743  -.00000250   .00000499   .00000074  -.00000009
2.5   .64550228   .03451047  -.00078887  -.00025434   .00000308   .00000558   .00000059  -.00000015
2.6   .67897859   .03347630  -.00103417  -.00024530   .00000904   .00000596   .00000038  -.00000021
2.7   .71119055   .03221196  -.00126434  -.00023018   .00001512   .00000608   .00000012  -.00000026
2.8   .74192900   .03073845  -.00147351  -.00020916   .00002102   .00000590  -.00000018  -.00000030
2.9   .77101120   .02908220  -.00165625  -.00018274   .00002642   .00000540  -.00000049  -.00000031
3.0   .79828544   .02727424  -.00180796  -.00015172   .00003103   .00000461  -.00000080  -.00000030
3.1   .82363457   .02534913  -.00192511  -.00011715   .00003457   .00000354  -.00000106  -.00000027
3.2   .84697828   .02334371  -.00200542  -.00008031   .00003684   .00000227  -.00000127  -.00000021
3.3   .86827397   .02129569  -.00204802  -.00004260   .00003770   .00000086  -.00000140  -.00000013
3.4   .88751616   .01924219  -.00205350  -.00000548   .00003712  -.00000058  -.00000144  -.00000004
3.5   .90473453   .01721837  -.00202382   .00002968   .00003516  -.00000196  -.00000139   .00000006
3.6   .91999072   .01525619  -.00196218   .00006164   .00003196  -.00000320  -.00000124   .00000015
3.7   .93337410   .01338339  -.00187280   .00008938   .00002774  -.00000422  -.00000102   .00000022
3.8   .94499685   .01162275  -.00176064   .00011217   .00002279  -.00000495  -.00000073   .00000028
3.9   .95498854   .00999169  -.00163106   .00012957   .00001741  -.00000538  -.00000043   .00000031
4.0   .96349065   .00850211  -.00148958   .00014148   .00001191  -.00000550  -.00000012   .00000031
4.1   .97065124   .00716059  -.00134152   .00014806   .00000658  -.00000533   .00000017   .00000029
4.2   .97662005   .00596881  -.00119178   .00014974   .00000167  -.00000491   .00000042   .00000025
4.3   .98154420   .00492414  -.00104467   .00014711  -.00000263  -.00000430   .00000061   .00000019
4.4   .98556459   .00402039  -.00090375   .00014092  -.00000619  -.00000357   .00000074   .00000013
4.5   .98881318   .00324859  -.00077180   .00013195  -.00000896  -.00000277   .00000080   .00000006
4.6   .99141100   .00259782  -.00065077   .00012103  -.00001093  -.00000197   .00000080   .00000001
4.7   .99346694   .00205594  -.00054188   .00010889  -.00001214  -.00000121   .00000076  -.00000005
4.8   .99507722   .00161027  -.00044566   .00009622  -.00001267  -.00000053   .00000068  -.00000008
4.9   .99632541   .00124819  -.00036208   .00008358  -.00001264   .00000003   .00000057  -.00000011
5.0   .99728295   .00095755  -.00029065   .00007144  -.00001215   .00000049   .00000045  -.00000011
5.1   .99800996   .00072701  -.00023054   .00006011  -.00001133   .00000082   .00000033  -.00000012
5.2   .99855625   .00054629  -.00018071   .00004982  -.00001028   .00000104   .00000022  -.00000011
5.3   .99896253   .00040628  -.00014001   .00004070  -.00000912   .00000116   .00000012  -.00000010
5.4   .99926158   .00029905  -.00010723   .00003278  -.00000792   .00000120   .00000004  -.00000008
5.5   .99947944   .00021786  -.00008119   .00002604  -.00000674   .00000118  -.00000003  -.00000007
5.6   .99963654   .00015709  -.00006077   .00002042  -.00000563   .00000111  -.00000007  -.00000004
5.7   .99974865   .00011212  -.00004498   .00001579  -.00000462   .00000101  -.00000010  -.00000004
5.8   .99982785   .00007920  -.00003292   .00001206  -.00000373   .00000089  -.00000012  -.00000001
5.9   .99988323   .00005533  -.00002382   .00000909  -.00000297   .00000076  -.00000013  -.00000001
6.0   .99992155   .00003832  -.00001705   .00000677  -.00000232   .00000065  -.00000012   .00000001
6.1   .99994781   .00002625  -.00001207   .00000498  -.00000179   .00000053  -.00000012   .00000000



6.2   .99996561   .00001780  -.00000845   .00000362  -.00000136   .00000043  -.00000010   .00000002
6.3   .99997756   .00001195  -.00000585   .00000260  -.00000102   .00000034  -.00000009   .00000001
6.4   .99998550   .00000794  -.00000401   .00000184  -.00000075   .00000027  -.00000007   .00000002
6.5   .99999072   .00000522  -.00000272   .00000129  -.00000055   .00000020  -.00000006   .00000001
6.6   .99999412   .00000340  -.00000182   .00000090  -.00000040   .00000016  -.00000005   .00000002
6.7   .99999630   .00000219  -.00000121   .00000061  -.00000028   .00000011  -.00000004   .00000000
6.8   .99999770   .00000140  -.00000079   .00000042  -.00000020   .00000009  -.00000003   .00000002
6.9   .99999858   .00000088  -.00000052   .00000028  -.00000014   .00000006  -.00000003   .00000000
7.0   .99999914   .00000055  -.00000033   .00000018  -.00000009   .00000004  -.00000001   .00000001
7.1   .99999948   .00000034  -.00000021   .00000012  -.00000006   .00000003  -.00000002  -.00000000
7.2   .99999969   .00000021  -.00000013   .00000008  -.00000004   .00000002  -.00000001   .00000001
7.3   .99999981   .00000013  -.00000008   .00000005  -.00000003   .00000001  -.00000001  -.00000000
7.4   .99999989   .00000008  -.00000005   .00000003  -.00000002   .00000001  -.00000000   .00000001
7.5   .99999993   .00000005  -.00000003   .00000002  -.00000001   .00000000  -.00000001  -.00000000
7.6   .99999996   .00000003  -.00000002   .00000001  -.00000001   .00000001   .00000000   .00000001
7.7   .99999998   .00000002  -.00000001   .00000001  -.00000001   .00000000  -.00000000  -.00000001
7.8   .99999998   .00000001  -.00000001   .00000000  -.00000000   .00000000   .00000000   .00000001
7.9   .99999999   .00000001  -.00000000   .00000000  -.00000000  -.00000000  -.00000000  -.00000001
```

Table IV. Milne Method Sample Solution No. 2.

```
FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    =  .7000000000
D2F(0) = 1.0598077730

 ETA          F            DF           D2F           D3F         PRED D2F       C3

 .000     .00000000    .00000000   1.05980777   -.70000000   1.05980777    .0E+00
 .100     .00518241    .10248264    .98988204   -.69777809    .98988204    .0E+00
 .200     .02026401    .19799111    .92039702   -.69121057    .92039702   -.1E-10
 .300     .04455030    .28659098    .85177962   -.68045296    .85177962    .3E-10
 .400     .07735545    .36838966    .78444020   -.66568307    .78444020   -.3E-10
 .500     .11800649    .44353468    .71876971   -.64711332    .71876971    .0E+00
 .600     .16584674    .51221152    .65513573   -.62499968    .65513573   -.1E-10
 .700     .22024042    .57464106    .59387780   -.59964725    .59387780    .0E+00
 .800     .28057512    .63107654    .53530242   -.57141222    .53530242    .1E-10
 .900     .34626530    .68179996    .47967797   -.54070001    .47967797   -.1E-10
1.000     .41675491    .72711807    .42723006   -.50795975    .42723006   -.7E-11
1.100     .49151962    .76735787    .37813752   -.47367534    .37813752    .0E+00
1.200     .57006861    .80286178    .33252957   -.43835374    .33252957   -.7E-11
1.300     .65194587    .83398259    .29048413   -.40251106    .29048413    .1E-10
1.400     .73673096    .86107830    .25202770   -.36665752    .25202770   -.7E-11
1.500     .82403930    .88450703    .21713665   -.33128225    .21713665    .1E-10
1.600     .91352192    .90462216    .18574013   -.29683881    .18574013   -.4E-11
1.700    1.00486476    .92176779    .15772425   -.26373245    .15772425    .1E-10
1.800    1.09778754    .93627468    .13293744   -.23230987    .13293744   -.7E-11
1.900    1.19204223    .94845684    .11119671   -.20285192    .11119671    .2E-10
2.000    1.28741125    .95860866    .09229446   -.17556954    .09229446   -.7E-11
2.100    1.38370540    .96700285    .07600553   -.15060310    .07600553    .2E-10
2.200    1.48076156    .97388902    .06209417   -.12802478    .06209417   -.7E-11
2.300    1.57844047    .97949294    .05032061   -.10784359    .05032061    .2E-10
2.400    1.67662418    .98401647    .04044706   -.09001262    .04044706   -.8E-11
2.500    1.77521374    .98763799    .03224284   -.07443777    .03224284    .3E-10
2.600    1.87412675    .99051335    .02548866   -.06098729    .02548866   -.7E-11
2.700    1.97329605    .99277721    .01997987   -.04950158    .01997987    .3E-10
2.800    2.07266585    .99454456    .01552877   -.03980273    .01552877   -.4E-11
2.900    2.17219167    .99591256    .01196601   -.03170319    .01196601    .4E-10
3.000    2.27183777    .99696234    .00914114   -.02501346    .00914114    .2E-11
3.100    2.37157578    .99776097    .00692251   -.01954838    .00692251    .4E-10
3.200    2.47138343    .99836324    .00519652   -.01513217    .00519652    .5E-11
3.300    2.57124337    .99881346    .00386655   -.01160202    .00386655    .5E-10
3.400    2.67114224    .99914704    .00285151   -.00881044    .00285151    .1E-10
3.500    2.77106983    .99939201    .00208423   -.00662648    .00208423    .6E-10
3.600    2.87101842    .99957030    .00150978   -.00493607    .00150978    .2E-10
3.700    2.97098224    .99969890    .00108384   -.00364153    .00108384    .8E-10
3.800    3.07095698    .99979083    .00077104   -.00266063    .00077104    .4E-10
3.900    3.17093951    .99985575    .00054354   -.00192520    .00054354    .1E-09
4.000    3.27092753    .99990165    .00037968   -.00137959    .00037968    .6E-10
4.100    3.37091938    .99993345    .00026280   -.00097905    .00026280    .1E-09
4.200    3.47091389    .99995535    .00018023   -.00068806    .00018023    .1E-09
4.300    3.57091022    .99997032    .00012247   -.00047887    .00012247    .2E-09
4.400    3.67090779    .99998044    .00008245   -.00033005    .00008245    .1E-09
4.500    3.77090620    .99998722    .00005499   -.00022526    .00005499    .2E-09
4.600    3.87090516    .99999173    .00003634   -.00015225    .00003634    .2E-09
4.700    3.97090449    .99999469    .00002379   -.00010190    .00002379    .3E-09
4.800    4.07090406    .99999663    .00001543   -.00006753    .00001543    .3E-09
4.900    4.17090379    .99999787    .00000991   -.00004432    .00000991    .4E-09
5.000    4.27090362    .99999867    .00000631   -.00002880    .00000631    .5E-09
5.100    4.37090352    .99999918    .00000398   -.00001854    .00000398    .6E-09
5.200    4.47090345    .99999949    .00000248   -.00001181    .00000248    .7E-09
5.300    4.57090341    .99999969    .00000154   -.00000745    .00000153    .9E-09
5.400    4.67090337    .99999981    .00000094   -.00000465    .00000094    .1E-08
5.500    4.77090337    .99999989    .00000057   -.00000288    .00000057    .1E-08
5.600    4.87090337    .99999993    .00000034   -.00000176    .00000034    .2E-08
5.700    4.97090336    .99999996    .00000020   -.00000106    .00000020    .2E-08
5.800    5.07090336    .99999998    .00000012   -.00000063    .00000012    .2E-08
5.900    5.17090335    .99999998    .00000007   -.00000037    .00000006    .3E-08
6.000    5.27090335    .99999999    .00000004   -.00000021    .00000003    .4E-08
```

Table V. Table of Differences for Sample Solution No. 2.

TABLE OF DIFFERENCES FOR

BETA = .7000000000

D2F(0) = 1.0598077730

ETA	DF	D1	D2	D3	D4	D5	D6	D7
.0	.00000000							
.1	.10248264	.10248264						
.2	.19799111	.09550848	-.00697416					
.3	.28659098	.08859986	-.00690861	.00006555				
.4	.36838966	.08179868	-.00680118	.00010743	.00004188			
.5	.44353468	.07514502	-.00665366	.00014752	.00004007	-.00000179		
.6	.51221152	.06867684	-.00646818	.00018548	.00003796	-.00000213	-.00000034	
.7	.57464106	.06242954	-.00624730	.00022088	.00003540	-.00000256	-.00000043	-.00000009
.8	.63107654	.05643548	-.00599407	.00025323	.00003234	-.00000305	-.00000049	-.00000006
.9	.68179996	.05072342	-.00571206	.00028201	.00002879	-.00000356	-.00000051	-.00000002
1.0	.72711807	.04531811	-.00540531	.00030675	.00002473	-.00000405	-.00000049	.00000002
1.1	.76735787	.04023980	-.00507831	.00032700	.00002025	-.00000448	-.00000043	.00000006
1.2	.80286178	.03550391	-.00473589	.00034242	.00001542	-.00000483	-.00000035	.00000009
1.3	.83398259	.03112081	-.00438310	.00035279	.00001036	-.00000506	-.00000023	.00000012
1.4	.86107830	.02709571	-.00402510	.00035800	.00000522	-.00000515	-.00000009	.00000013
1.5	.88450703	.02342873	-.00366698	.00035813	.00000013	-.00000509	.00000006	.00000015
1.6	.90462216	.02011513	-.00331360	.00035337	-.00000475	-.00000488	.00000021	.00000015
1.7	.92176779	.01714563	-.00296950	.00034410	-.00000928	-.00000452	.00000036	.00000015
1.8	.93627468	.01450690	-.00263873	.00033077	-.00001332	-.00000404	.00000048	.00000012
1.9	.94845684	.01218216	-.00232474	.00031399	-.00001678	-.00000346	.00000059	.00000011
2.0	.95860866	.01015182	-.00203034	.00029440	-.00001959	-.00000281	.00000065	.00000007
2.1	.96700285	.00839419	-.00175763	.00027271	-.00002170	-.00000211	.00000070	.00000004
2.2	.97388902	.00688617	-.00150802	.00024960	-.00002310	-.00000141	.00000070	.00000000
2.3	.97949294	.00560392	-.00128225	.00022578	-.00002383	-.00000073	.00000068	-.00000002
2.4	.98401647	.00452353	-.00108040	.00020185	-.00002393	-.00000010	.00000063	-.00000005
2.5	.98763799	.00362152	-.00090201	.00017839	-.00002346	.00000046	.00000056	-.00000007
2.6	.99051335	.00287537	-.00074615	.00015586	-.00002253	.00000093	.00000047	-.00000009
2.7	.99277721	.00226386	-.00061151	.00013464	-.00002122	.00000131	.00000038	-.00000009
2.8	.99454456	.00176735	-.00049651	.00011501	-.00001963	.00000159	.00000028	-.00000010
2.9	.99591256	.00136799	-.00039936	.00009715	-.00001786	.00000177	.00000019	-.00000009
3.0	.99696234	.00104979	-.00031821	.00008115	-.00001599	.00000187	.00000009	-.00000009
3.1	.99776097	.00079863	-.00025115	.00006705	-.00001410	.00000189	.00000002	-.00000007
3.2	.99836324	.00060227	-.00019636	.00005480	-.00001226	.00000185	-.00000005	-.00000007
3.3	.99881346	.00045021	-.00015206	.00004430	-.00001050	.00000176	-.00000009	-.00000004
3.4	.99914704	.00033358	-.00011664	.00003542	-.00000887	.00000162	-.00000013	-.00000004
3.5	.99939201	.00024497	-.00008861	.00002803	-.00000740	.00000148	-.00000015	-.00000002
3.6	.99957030	.00017829	-.00006668	.00002193	-.00000609	.00000131	-.00000017	-.00000002
3.7	.99969890	.00012860	-.00004969	.00001699	-.00000495	.00000114	-.00000017	.00000000
3.8	.99979083	.00009193	-.00003668	.00001301	-.00000397	.00000098	-.00000017	-.00000000
3.9	.99985595	.00006512	-.00002681	.00000987	-.00000315	.00000083	-.00000015	.00000002
4.0	.99990165	.00004571	-.00001941	.00000740	-.00000247	.00000068	-.00000014	.00000001
4.1	.99993345	.00003179	-.00001392	.00000549	-.00000191	.00000056	-.00000012	.00000002
4.2	.99995535	.00002191	-.00000988	.00000403	-.00000146	.00000045	-.00000011	.00000001
4.3	.99997032	.00001496	-.00000695	.00000293	-.00000110	.00000036	-.00000009	.00000002
4.4	.99998044	.00001012	-.00000484	.00000211	-.00000082	.00000028	-.00000008	.00000001
4.5	.99998722	.00000678	-.00000334	.00000150	-.00000061	.00000022	-.00000006	.00000002
4.6	.99999173	.00000451	-.00000228	.00000106	-.00000044	.00000016	-.00000005	.00000001
4.7	.99999469	.00000296	-.00000154	.00000074	-.00000032	.00000012	-.00000004	.00000001
4.8	.99999663	.00000193	-.00000103	.00000051	-.00000023	.00000009	-.00000003	.00000001
4.9	.99999787	.00000125	-.00000068	.00000035	-.00000016	.00000007	-.00000002	.00000001
5.0	.99999867	.00000080	-.00000045	.00000023	-.00000011	.00000005	-.00000002	.00000000
5.1	.99999918	.00000051	-.00000029	.00000016	-.00000008	.00000004	-.00000001	.00000001
5.2	.99999949	.00000032	-.00000019	.00000010	-.00000005	.00000002	-.00000001	.00000000
5.3	.99999969	.00000020	-.00000012	.00000007	-.00000004	.00000002	-.00000001	.00000001
5.4	.99999981	.00000012	-.00000008	.00000004	-.00000002	.00000001	-.00000001	-.00000000
5.5	.99999989	.00000007	-.00000005	.00000003	-.00000002	.00000001	-.00000000	.00000000
5.6	.99999993	.00000004	-.00000003	.00000002	-.00000001	.00000001	-.00000000	-.00000000
5.7	.99999996	.00000003	-.00000002	.00000001	-.00000001	.00000000	-.00000000	.00000000
5.8	.99999998	.00000002	-.00000001	.00000001	-.00000000	.00000000	-.00000000	-.00000000
5.9	.99999998	.00000001	-.00000001	.00000000	-.00000000	.00000000	-.00000000	.00000000
6.0	.99999999	.00000001	-.00000000	.00000000	-.00000000	.00000000	-.00000000	-.00000000

Milne Method solutions and the Runge-Kutta solutions indicates that both solutions are accurate to seven places of decimals.

7. Comments on the Methods of Solution.

As will be noted from the descriptions of the two methods given in Appendices B and C, the Runge-Kutta Method is self-starting, that is, it requires only the values at the first point to be given, whereas the Milne Five-Point Method requires six starting points. This property of the Milne Method requires that a rather cumbersome set of equations be used to obtain the six starting points before the Five-Point Method proper can be used. However, this presents no difficulty when using a high-speed digital computer. As has been pointed out, the Milne Method produces a comparable solution with a considerably larger step-width than that which can be used with the Runge-Kutta Method (0.01 versus 0.001) and this means that the Milne Method produces a solution in approximately one-tenth the time required by the Runge-Kutta Method. Starting with an initial estimate of $f''(0) = 0.0$, the Milne Method requires from three to six minutes computing time on the CDC 1604 Computer. This difference in running time is due primarily to the method which was used to alter the initial estimate of $f''(0)$; namely, the value of $f''(0)$ was altered by a fixed increment in one particular decimal position at a time, starting with the first decimal place and moving one place at a time to subsequent decimal places as the solution was built up.

An attempt was made to let the solutions proceed beyond the cut-off point once the solution to the cut-off point had been found; that is, to let the solution continue asymptotically to higher values of η. It was found that this could be accomplished with the Runge-

Kutta Method but that it could not be done with the Milne Method. Beyond the cut-off point, the Milne Method would proceed asymptotically for only a short distance and then it would depart extremely rapidly away from the asymptote. An attempt to rectify this property of the Milne Method was not pursued. It appears that if it were desired to carry a solution beyond the cut-off point, as might well be the case when dealing with the case of compressible fluids, that the most advantageous way would be to determine the solution to the cut-off point by the Milne Method and then introduce this solution into the Runge-Kutta Method.

8. Conclusion.

The results quoted in this report confirm the solutions of the Falkner and Skan equation given by Hartree. The methods of solution described herein are particularly suitable for use with a modern digital computer and the results produced are considered to be accurate to seven places of decimals.

Extension of the methods described to the appropriate case of compressible flow should present little difficulty.

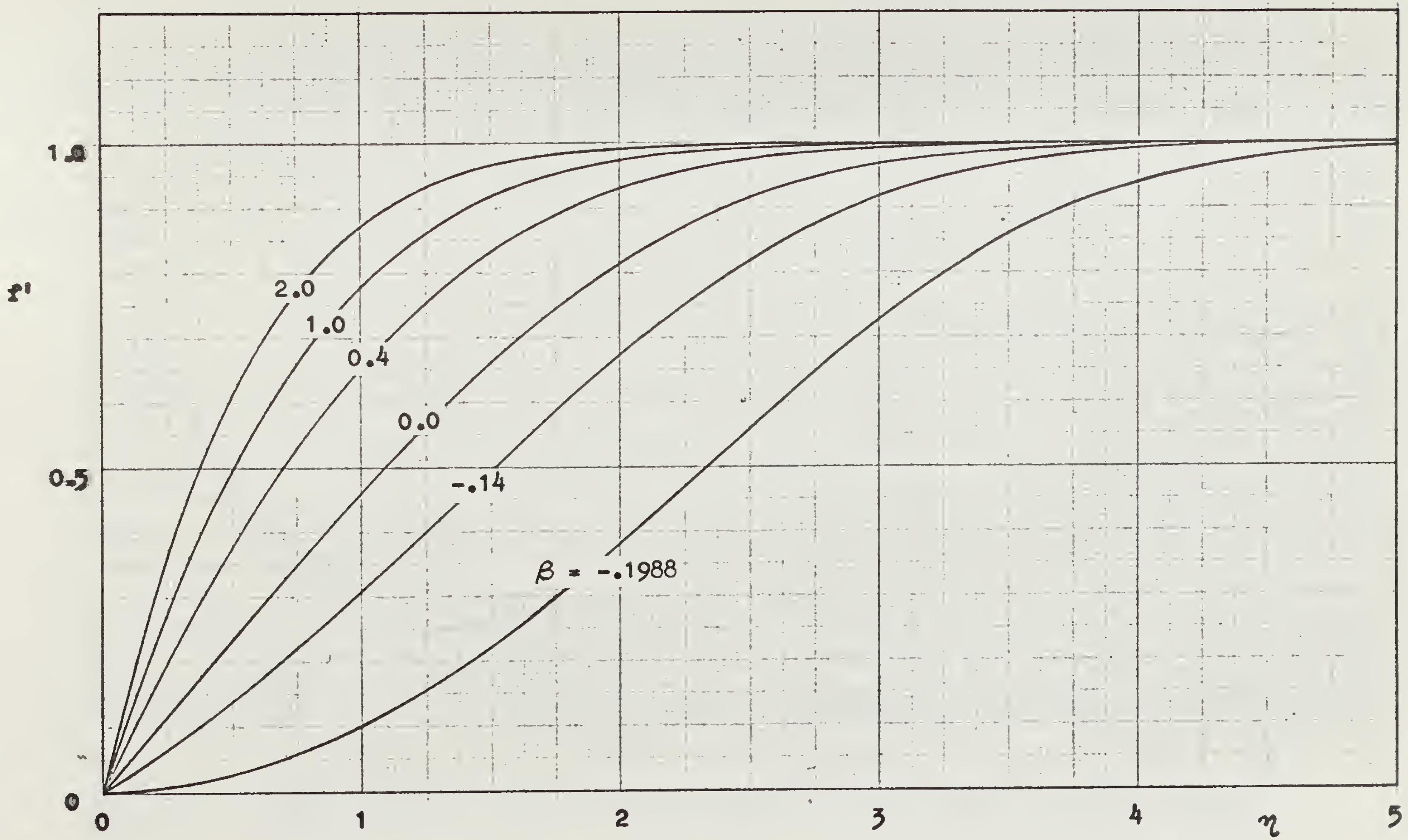

Figure 1. Graph showing f' versus η for solutions of the Falkner and Skan equation for values of β as indicated.

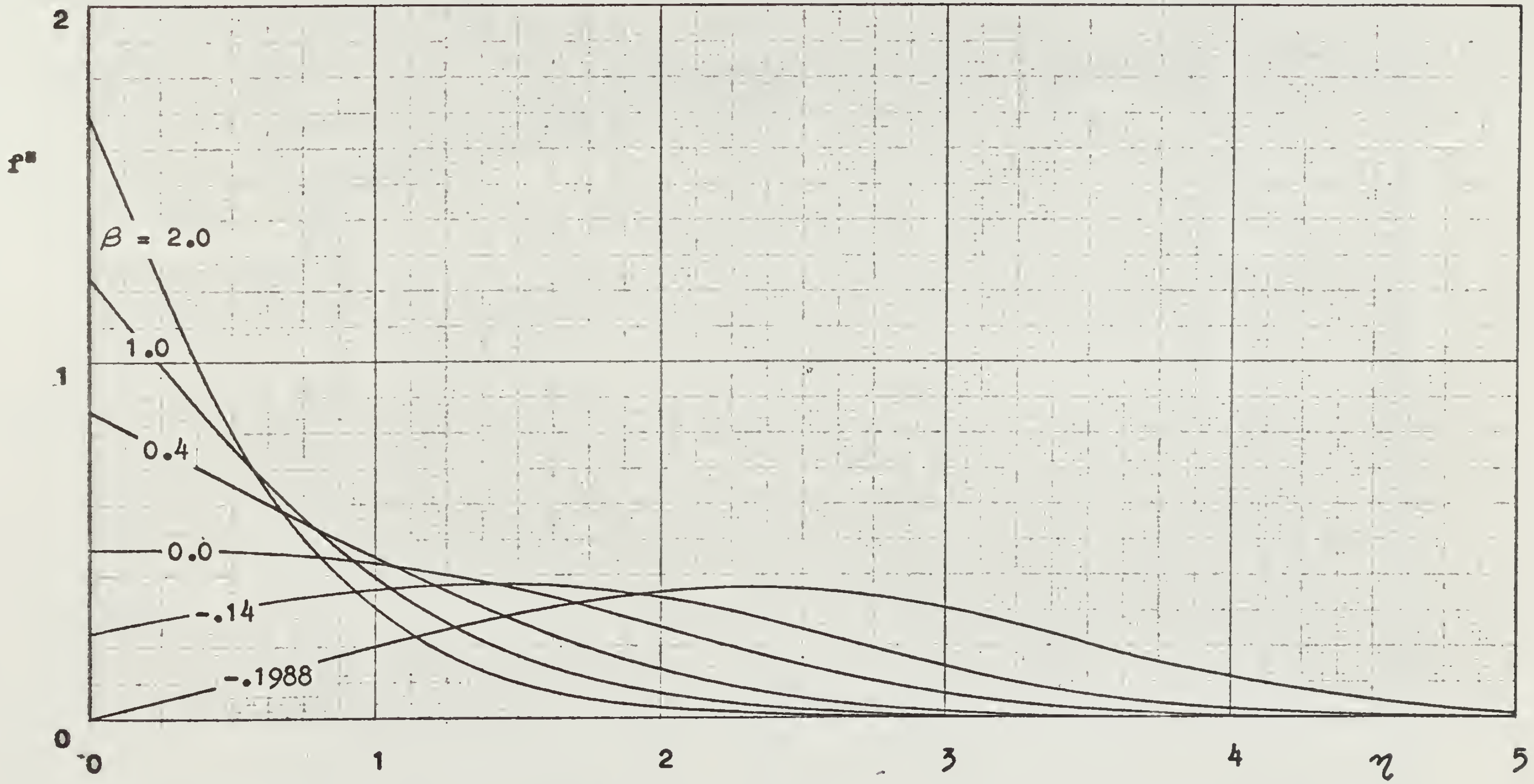

Figure 2. Graph showing f" versus η for solutions of the Falkner and Skan equation for values of β as indicated.

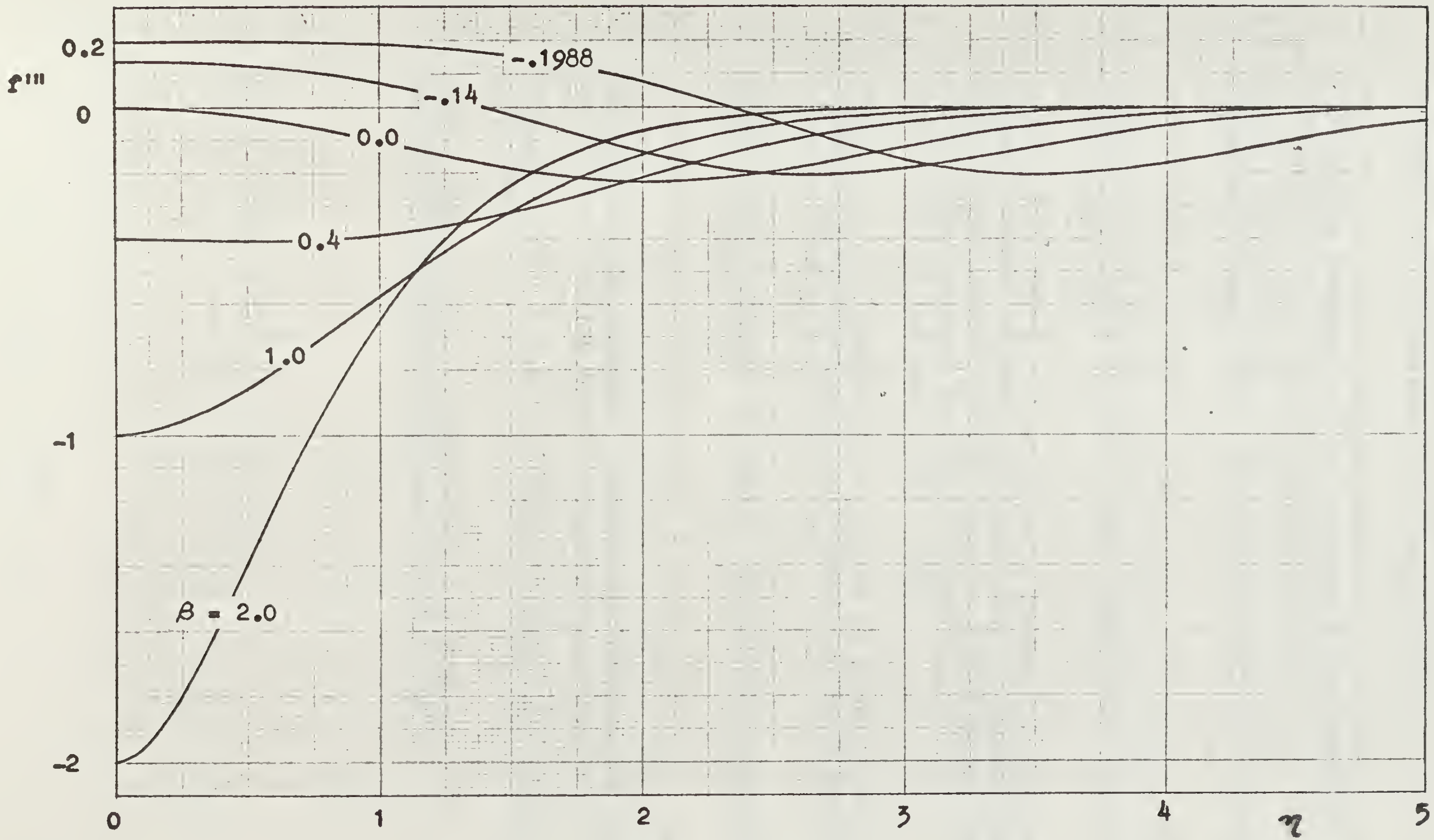

Figure 3. Graph showing f''' versus η for solutions of the Falkner and Skan equation for values of β as indicated.

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APPENDIX A

THE BOUNDARY LAYER EQUATIONS

The complete description of any general fluid motion requires a complex system of equations consisting of:

1. the Navier-Stokes equations,
2. the continuity equation,
3. the equation of state, and
4. the energy equation.

In the case of incompressible fluids, the equation of state and the energy equation are not required, and the system of equations is reduced to the Navier-Stokes equations and the continuity equation. Furthermore, these latter equations are in themselves somewhat simplified when the fluid is incompressible. The Navier-Stokes equations for an incompressible fluid are:

$$\left.\begin{aligned}
&\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \\
&\qquad X - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \\
&\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \\
&\qquad Y - \frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \\
&\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = \\
&\qquad Z - \frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)
\end{aligned}\right\} \quad \text{(A-1)}$$

and the continuity equation is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \text{(A-2)}$$

This report is concerned with the case of two-dimensional, steady, incompressible flow. Furthermore, it is assumed that the body forces are negligible. The system of equations (A-1) and (A-2) then becomes:

$$\left.\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\ u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \end{aligned}\right\} \qquad \text{(A-3)}$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(A-4)}$$

with the boundary conditions:

$u = 0$, $v = 0$, at $y = 0$ and $u \rightarrow u_1$ as $y \rightarrow \infty$.

Prandtl further simplified these equations by considering the order of magnitude of each term and then neglecting the smaller terms. Before proceeding, it is convenient to express the equations (A-3) and (A-4) in terms of non-dimensional quantities by referring the linear dimensions, x and y, to a characteristic length, L, of the body about which the fluid is flowing and by referring the velocities, u, v, and u_1, to the free-stream velocity, U_0. The kinematic viscosity, ν, is related to a characteristic Reynolds number, R_e, namely,

$$\frac{1}{R_e} = \frac{\nu}{U_0 L} ,$$

and the pressure, p, is made dimensionless by referring it to the quantity ρU_0^2. The same symbols will be used for the dimensionless quantities as for their dimensional counterparts.

Now, rewriting equations (A-3) and (A-4) in dimensionless form

gives:

$$\left.\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\ u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \end{aligned}\right\} \qquad \text{(A-5)}$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(A-6)}$$

with the boundary conditions:

$u = 0$, $v = 0$, at $y = 0$ and $u \to u_1$ as $y \to \infty$.

In order to compare magnitudes, it is assumed that the boundary layer thickness, δ , is very small compared to the characteristic length, L , i.e.,

$$\delta \ll L \quad ,$$

and therefore the dimensionless boundary layer thickness, δ/L , which will be denoted δ_1 , is very small compared to unity, i.e.,

$$\delta_1 \ll 1 \; .$$

Further, it may be shown [4] in several exact solutions of the Navier-Stokes equations that the boundary layer thickness is proportional to the square root of the kinematic viscosity. Thus $1/Re$ is of the order of δ_1^2 . It is also assumed that u_1 and x are of the order 1, i.e., $u \sim u_1 \sim 1$ and $x \sim 1$. Thus $\partial u/\partial x \sim 1$ and from the continuity equation (A-6) it is seen that $\partial v/\partial y \sim 1$ but $\partial y \sim \delta_1$ so that $\partial v \sim \delta_1$ and therefore $v \sim \delta_1$. Furthermore, $\partial^2 u/\partial x^2 \sim 1$, $\partial u/\partial y \sim 1/\delta_1$, $\partial^2 u/\partial y^2 \sim 1/\delta_1^2$, $\partial v/\partial x \sim \delta_1$, $\partial^2 v/\partial x^2 \sim \delta_1$, and $\partial^2 v/\partial y^2 \sim \delta_1/\delta_1^2 \sim 1/\delta_1$.

Rewriting equations (A-5) and (A-6) with the orders of magnitude written below each term enables a comparison to be made:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad \text{(A-5a)}$$

$$1 \quad 1 \qquad \delta_1 \quad \frac{1}{\delta_1} \qquad\qquad \delta_1^2 \quad 1 \qquad \frac{1}{\delta_1^2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \qquad \text{(A-5b)}$$

$$1 \quad \delta_1 \qquad \delta_1 \quad 1 \qquad\qquad \delta_1^2 \quad \delta_1 \qquad \frac{1}{\delta_1}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(A-6)}$$

$$1 \qquad 1$$

Now, if the terms of order δ_1 or less are neglected, equations (A-5) and (A-6) are reduced to:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} \qquad \text{(A-7)}$$

$$0 = -\frac{\partial p}{\partial y} \qquad \text{(A-8)}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(A-9)}$$

with the boundary conditions:

$u=0,\ v=0$, at $y=0$ and $u \to u_1$ as $y \to \infty$.

Equation (A-8) indicates that within the boundary layer, the pressure gradient normal to the boundary is negligible. Therefore, since $\partial p/\partial y = 0$, $\partial p/\partial x = dp/dx$ and equation (A-7) can be written:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} \qquad \text{(A-10)}$$

Equation (A-10) is the non-dimensional form of the two-dimensional, steady, incompressible flow boundary layer equation. In the dimensional form it is:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \ . \qquad \text{(A-11)}$$

The Falkner and Skan boundary layer equation will now be derived in the non-dimensional form from equation (A-10). Falkner and Skan [1] assumed that the velocity, u_1, at the outer limit of the boundary layer is proportional to some power, m, of the length coordinate tangential to the boundary, measured from the forward stagnation point, i.e.,

$$u_1 = a x^{m} \tag{A-12}$$

where a is a constant.

At the outer limit of the boundary layer, $u = u_1$ and therefore, from equation (A-10),

$$u_1 \frac{\partial u_1}{\partial x} = - \frac{dp}{dx} \quad , \tag{A-13}$$

and, noting that $\partial u_1/\partial x = du_1/dx$, equation (A-10) becomes:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_1 \frac{du_1}{dx} + \frac{1}{Re} \frac{\partial^2 u}{\partial y^2} \quad . \tag{A-14}$$

Now define a non-dimensional boundary layer coordinate, η, and a non-dimensional stream function, ψ, such that:

$$\eta = y \sqrt{\frac{m+1}{2} \cdot \frac{u_1 Re}{x}} = y \sqrt{\frac{m+1}{2} \cdot a Re} \; x^{\frac{m-1}{2}} \tag{A-15}$$

and

$$\psi = \sqrt{\frac{2}{m+1} \cdot \frac{u_1 x}{Re}} \, f = \sqrt{\frac{2}{m+1} \cdot \frac{a}{Re}} \; x^{\frac{m+1}{2}} f \tag{A-16}$$

where $f = f(\eta)$, a function of η only. The stream function ψ

satisfies the continuity equation, (A-9), namely:

$$u = \frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\partial \psi}{\partial x} \quad . \qquad \text{(A-17)}$$

The components of equation (A-14) may now be obtained from equations (A-12), (A-15), (A-16), and (A-17), namely:

$$u = u_1 f' \quad , \qquad \text{(A-18)}$$

$$v = -\frac{m+1}{2} \cdot \frac{\psi}{x} - \frac{m-1}{2} \cdot \frac{y u_1}{x} \cdot f' \quad , \qquad \text{(A-19)}$$

$$\frac{\partial u}{\partial x} = \frac{m-1}{2} \cdot \frac{u_1 \eta}{x} \cdot f'' + m \frac{u_1}{x} \cdot f' \quad , \qquad \text{(A-20)}$$

$$\frac{\partial u}{\partial y} = \frac{u_1 \eta}{y} \cdot f'' \quad , \qquad \text{(A-21)}$$

$$\frac{\partial^2 u}{\partial y^2} = u_1 \left(\frac{\eta}{y}\right)^2 f''' \quad \text{, and} \qquad \text{(A-22)}$$

$$u_1 \frac{du_1}{dx} = m \frac{u_1^2}{x} \quad . \qquad \text{(A-23)}$$

where the primes denote differentiation with respect to η.

Substitution of these components into equation (A-14) yields, after simplification:

$$\frac{m+1}{2} f''' + \frac{m+1}{2} f f'' + m (1 - f'^2) = 0 \quad . \qquad \text{(A-24)}$$

Multiply through by $\frac{2}{m+1}$ and define

$$\beta = \frac{2m}{m+1} \quad . \qquad \text{(A-25)}$$

Equation (A-24) then becomes:

$$f''' + f f'' + \beta(1 - f'^2) = 0 \qquad \text{(A-26)}$$

with the boundary conditions:

$$f = 0,\ f' = 0,\ \text{at}\ \eta = 0,\ \text{and}\ f' \to 1\ \text{as}\ \eta \to \infty\ .$$

Equation (A-26) is the Falkner and Skan boundary layer equation with which this report is concerned.

APPENDIX B

PROGRAMMING THE MILNE METHOD NUMERICAL SOLUTION OF THE FALKNER AND SKAN EQUATION FOR THE DIGITAL COMPUTER

B1. The Equation.

For the purpose of notation in this section, the variables η and f will be denoted by x and y respectively. The Falkner and Skan equation then becomes:

$$y''' + y\, y'' + \beta(1 - y'^2) = 0 \tag{B-1}$$

where y is a function of x only and the primes denote differentiation with respect to x. The boundary conditions to be imposed are:

$$y = 0,\ y' = 0,\ \text{at } x = 0, \tag{B-2}$$

$$y' \rightarrow 1 \text{ from below as } x \rightarrow \infty, \text{ and} \tag{B-3}$$

$$y'' \rightarrow 0 \text{ from above, } y''' \rightarrow 0 \text{ from below, as } x \rightarrow \infty . \tag{B-4}$$

B2. Description of the Milne Method.

The application of the "Milne Five-Point Method" [8],[9] to the numerical solution of equation (B-1) is illustrated below.

Rewrite the differential equation (B-1) as:

$$y_n''' = -y_n\, y_n'' - \beta(1 - y_n'^2) \tag{B-5}$$

The initial conditions are:

$$y_o = y(0) = 0, \tag{B-6}$$

$$y_o' = y'(0) = 0, \tag{B-7}$$

$$y_o'' = y''(0) = \text{an ASSUMED value}, \tag{B-8}$$

$$\beta = \text{a SPECIFIED value.} \tag{B-9}$$

From equation (B-5):

$$y_0''' = y'''(0) = -\beta, \tag{B-10}$$

$$y_0^{iv} = -y_0\, y_0''' - (1 - 2\beta) y_0'\, y_0'' = 0. \tag{B-11}$$

A suitable step-width,

$$h = x_{n+1} - x_n \,, \tag{B-12}$$

must also be selected. The selection of a suitable value of h has already been discussed on page 5.

STEP 1.: Estimate values of y_{-1}''' and y_1'''.

Milne suggests:

$$y_{-1}''' = y_0''' - h\, y_0^{iv} \,, \text{ and} \tag{B-13}$$

$$y_1''' = y_0''' + h\, y_0^{iv} \,. \tag{B-14}$$

In order to obtain a better estimate, the writer preferred to extend these equations by means of the Taylor Series expansion to:

$$y_{-1}''' = y_0''' - h\, y_0^{iv} + \frac{h^2}{2} y_0^{v} - \frac{h^3}{6}\, y_0^{vi} \,, \text{ and} \tag{B-15}$$

$$y_1''' = y_0''' + h\, y_0^{iv} + \frac{h^2}{2}\, y_0^{v} + \frac{h^3}{6}\, y_0^{vi} \,. \tag{B-16}$$

Equation (B-11) gives the expression for y_0^{iv}. Subsequent differentiation yields:

$$y_0^{v} = -y_0\, y_0^{iv} - 2(1 - \beta) y_0'\, y_0''' - (1 - 2\beta) {y_0''}^2 \,, \text{ and} \tag{B-17}$$

$$y_0^{vi} = -y_0\, y_0^{v} - (3 - 2\beta) y_0'\, y_0^{iv} - (4 - 6\beta) y_0''\, y_0''' \,. \tag{B-18}$$

Substitution of (B-17) and (B-18) and the initial conditions into (B-15) and (B-16) yields, after simplification,

$$y_{-1}''' = -\beta + \frac{h^2}{2}(2\beta - 1){y_0''}^2 - \frac{h^3}{3}\beta(2 - 3\beta) y_0'' \,, \text{ and} \tag{B-19}$$

$$y_1''' = -\beta + \frac{h^2}{2}(2\beta - 1){y_0''}^2 + \frac{h^3}{3}\beta(2 - 3\beta) y_0'' \,. \tag{B-20}$$

Evaluation of equations (B-19) and (B-20) will constitute Step 1.

STEP 2.: Estimate values of y''_{-1}, y''_1, y'_{-1}, y'_1, y_{-1}, and y_1 in that order as follows:

$$y''_{-1} = y''_0 - \frac{h}{24}(7y'''_{-1} + 16y'''_0 + y'''_1) + \frac{h^2}{4} y^{iv}_0 \, , \tag{B-21}$$

$$y''_1 = y''_0 + \frac{h}{24}(y'''_{-1} + 16y'''_0 + 7y'''_1) + \frac{h^2}{4} y^{iv}_0 \, , \tag{B-22}$$

$$y'_{-1} = y'_0 - \frac{h}{24}(7y''_{-1} + 16y''_0 + y''_1) + \frac{h^2}{4} y'''_0 \, , \tag{B-23}$$

$$y'_1 = y'_0 + \frac{h}{24}(y''_{-1} + 16y''_0 + 7y''_1) + \frac{h^2}{4} y'''_0 \, , \tag{B-24}$$

$$y_{-1} = y_0 - \frac{h}{24}(7y'_{-1} + 16y'_0 + y'_1) + \frac{h^2}{4} y''_0 \, , \text{ and} \tag{B-25}$$

$$y_1 = y_0 + \frac{h}{24}(y'_{-1} + 16y'_0 + 7y'_1) + \frac{h^2}{4} y''_0 \, . \tag{B-26}$$

An indication of the accuracy of these equations may be obtained from the remainder terms on the right-hand side of (B-25) and (B-26) which are, respectively, $R_{25} = +\frac{1}{180} h^5 y^v$ and $R_{26} = -R_{25}$, where y^v is evaluated at some point in the interval between x_{-1} and x_1. Evaluation of the remainder terms will not be carried out in this solution.

STEP 3.: Evaluate y'''_{-1} and y'''_1 from the differential equation, (B-5).

STEP 4.: Using the values of y'''_{-1} and y'''_1 from Step 3, repeat Steps 2 and 3 until no change, to the desired number of significant figures, occurs in the calculated values. At this point, the writer required agreement to within 1×10^{-8}.

STEP 5.: Estimate values of y''_2, y'_2, and y_2 as follows:

$$y''_2 = y''_0 + \frac{2h}{3}(-y'''_{-1} - y'''_0 + 5y'''_1) - 2h^2 y^{iv}_0 \, , \tag{B-27}$$

$$y'_2 = y'_o + \frac{h}{3}(y''_o + 4y''_1 + y''_2) \text{ , and} \qquad (B\text{-}28)$$

$$y_2 = y_o + \frac{h}{3}(y'_o + 4y'_1 + y'_2) \text{ .} \qquad (B\text{-}29)$$

STEP 6.: Evaluate y'''_2 from the differential equation, (B-5).

STEP 7.: Correct the values of y''_2, y'_2, and y_2 as follows:

$$y''_2 = y''_o + \frac{h}{3}(y'''_o + 4y'''_1 + y'''_2) \text{ ,} \qquad (B\text{-}30)$$

$$y'_2 = y'_o + \frac{h}{3}(y''_o + 4y''_1 + y''_2) \text{ , and} \qquad (B\text{-}28)$$

$$y_2 = y_o + \frac{h}{3}(y'_o + 4y'_1 + y'_2) \text{ .} \qquad (B\text{-}29)$$

STEP 8.: Evaluate y'''_2 from the differential equation, (B-5).

STEP 9.: Using the value of y'''_2 from Step 8, repeat Steps 7 and 8 until no change, to the desired number of significant figures, occurs in the calculated values. Here the writer required agreement to within 1×10^{-8}.

STEP 10.: Estimate values of y''_{-2}, y'_{-2}, and y_{-2} as follows:

$$y''_{-2} = y''_o - \frac{2h}{3}(5y'''_{-1} - y'''_o - y'''_1) - 2h^2\, y^{iv}_o \text{ ,} \qquad (B\text{-}31)$$

$$y'_{-2} = y'_o - \frac{h}{3}(y''_{-2} + 4y''_{-1} + y''_o) \text{ , and} \qquad (B\text{-}32)$$

$$y_{-2} = y_o - \frac{h}{3}(y'_{-2} + 4y'_{-1} + y'_o) \text{ .} \qquad (B\text{-}33)$$

STEP 11.: Evaluate y'''_{-2} from the differential equation, (B-5).

STEP 12.: Correct the values of y''_{-2}, y'_{-2}, and y_{-2} as follows:

$$y''_{-2} = y''_o - \frac{h}{3}(y'''_{-2} + 4y'''_{-1} + y'''_o) \text{ ,} \qquad (B\text{-}34)$$

$$y'_{-2} = y'_o - \frac{h}{3}(y''_{-2} + 4y''_{-1} + y''_o) \text{ , and} \qquad (B\text{-}32)$$

$$y_{-2} = y_o - \frac{h}{3}(y'_{-2} + 4y'_{-1} + y'_o) \ . \qquad \text{(B-33)}$$

STEP 13.: Evaluate y'''_{-2} from the differential equation, (B-5).

STEP 14.: Using the value of y'''_{-2} from Step 13, repeat Steps 12 and 13 until no change, to the desired number of significant figures, occurs in the calculated values. Here also, the writer required agreement to within 1×10^{-8}.

The remainder terms on the right-hand side of equations (B-27), (B-30), (B-31), and (B-34) are, respectively, $R_{27} = + \frac{7}{45}h^7\ y^{vii}$, $R_{30} = - \frac{1}{90}h^7\ y^{vii}$, $R_{31} = - R_{27}$, and $R_{34} = - R_{30}$.

STEP 15.: Now regard the values calculated by the previous steps as first approximations and refine them as follows:

$$y''_{-2} = (y''_o - \frac{4h}{15}y'''_o - \frac{h^2}{3}\ y^{iv}_o)$$
$$- \frac{h}{180}(53y'''_{-2} + 288y'''_{-1} - 32y'''_1 + 3y'''_2) \ , \qquad \text{(B-35)}$$

$$y'_{-2} = (y'_o - \frac{4h}{15}y''_o - \frac{h^2}{3}\ y'''_o)$$
$$- \frac{h}{180}(53y''_{-2} + 288y''_{-1} - 32y''_1 + 3y''_2) \ , \text{ and} \qquad \text{(B-36)}$$

$$y_{-2} = (y_o - \frac{4h}{15}\ y'_o - \frac{h^2}{3}\ y''_o)$$
$$- \frac{h}{180}(53y'_{-2} + 288y'_{-1} - 32y'_1 + 3y'_2) \ . \qquad \text{(B-37)}$$

Evaluate y'''_{-2} from equation (B-5).

$$y''_{-1} = (y''_o - \frac{19h}{30}\ y'''_o + \frac{11h^2}{48}\ y^{iv}_o)$$
$$- \frac{h}{2880}(- 21y'''_{-2} + 944y'''_{-1} + 144y'''_1 - 11y'''_2) \ , \qquad \text{(B-38)}$$

$$y'_{-1} = \left(y'_0 - \frac{19h}{30}\,y''_0 + \frac{11h^2}{48}\,y'''_0\right) - \frac{h}{2880}\left(-21y''_{-2} + 944y''_{-1} + 144y''_1 - 11y''_2\right), \text{ and} \qquad (B\text{-}39)$$

$$y_{-1} = \left(y_0 - \frac{19h}{30}\,y'_0 + \frac{11h^2}{48}\,y''_0\right) - \frac{h}{2880}\left(-21y'_{-2} + 944y'_{-1} + 144y'_1 - 11y'_2\right). \qquad (B\text{-}40)$$

Evaluate y'''_{-1} from equation (B-5).

$$y''_1 = \left(y''_0 + \frac{19h}{30}\,y'''_0 + \frac{11h^2}{48}\,y^{iv}_0\right) + \frac{h}{2880}\left(-11y'''_{-2} + 144y'''_{-1} + 944y'''_1 - 21y'''_2\right), \qquad (B\text{-}41)$$

$$y'_1 = \left(y'_0 + \frac{19h}{30}\,y''_0 + \frac{11h^2}{48}\,y'''_0\right) + \frac{h}{2880}\left(-11y''_{-2} + 144y''_{-1} + 944y''_1 - 21y''_2\right), \text{ and} \qquad (B\text{-}42)$$

$$y_1 = \left(y_0 + \frac{19h}{30}\,y'_0 + \frac{11h^2}{48}\,y''_0\right) + \frac{h}{2880}\left(-11y'_{-2} + 144y'_{-1} + 944y'_1 - 21y'_2\right). \qquad (B\text{-}43)$$

Evaluate y'''_1 from equation (B-5).

$$y''_2 = \left(y''_0 + \frac{4h}{15}\,y'''_0 - \frac{h^2}{3}\,y^{iv}_0\right) + \frac{h}{180}\left(3y'''_{-2} - 32y'''_{-1} + 288y'''_1 + 53y'''_2\right), \qquad (B\text{-}44)$$

$$y'_2 = \left(y'_0 + \frac{4h}{15}\,y''_0 - \frac{h^2}{3}\,y'''_0\right) + \frac{h}{180}\left(3y''_{-2} - 32y''_{-1} + 288y''_1 + 53y''_2\right), \text{ and} \qquad (B\text{-}45)$$

$$y_2 = \left(y_0 + \frac{4h}{15}\,y'_0 - \frac{h^2}{3}\,y''_0\right) + \frac{h}{180}\left(3y'_{-2} - 32y'_{-1} + 288y'_1 + 53y'_2\right). \qquad (B\text{-}46)$$

Evaluate y_2''' from the differential equation, (B-5).

STEP 16.: Repeat Step 15 until no change, to the desired number of significant figures, occurs in the calculated values. At this stage, and at all succeeding points, the writer required agreement to within 1 x 10^{-10}.

The remainder terms on the right-hand side of equations (B-35), (B-38), (B-41), and (B-44) are, respectively, $R_{35} = + \frac{32}{7560} h^7 y^{ix}$, $R_{38} = - \frac{5}{7560} h^7 y^{ix}$, $R_{41} = - R_{38}$, and $R_{44} = - R_{35}$.

STEP 17.: Predict y_3'', y_3', and y_3 as follows:

$$y_3'' = y_0'' + \frac{3h}{320}(- 49y_{-2}''' + 496y_{-1}''' + 416y_0''' - 944y_1''' + 401y_2''') + \frac{153h^2}{16} y_0^{iv} , \qquad (B\text{-}47)$$

$$y_3' = y_{-1}' + \frac{2h}{45}(7y_{-1}'' + 32y_0'' + 12y_1'' + 32y_2'' + 7y_3'') , \text{ and} \qquad (B\text{-}49a)$$

$$y_3 = y_{-1} + \frac{2h}{45}(7y_{-1}' + 32y_0' + 12y_1' + 32y_2' + 7y_3') . \qquad (B\text{-}50a)$$

STEP 18.: Evaluate y_3''' from the differential equation, (B-5).

STEP 19.: Correct the values of y_3'', y_3', and y_3 as follows:

$$y_3'' = y_{-1}'' + \frac{2h}{45}(7y_{-1}''' + 32y_0''' + 12y_1''' + 32y_2''' + 7y_3''') , \qquad (B\text{-}51a)$$

$$y_3' = y_{-1}' + \frac{2h}{45}(7y_{-1}'' + 32y_0'' + 12y_1'' + 32y_2'' + 7y_3'') , \text{ and} \qquad (B\text{-}49a)$$

$$y_3 = y_{-1} + \frac{2h}{45}(7y_{-1}' + 32y_0' + 12y_1' + 32y_2' + 7y_3') . \qquad (B\text{-}50a)$$

STEP 20.: Evaluate y_3''' from equation (B-5).

STEP 21.: Repeat Steps 19 and 20 until no change, to the desired number of significant figures, occurs in the calculated values.

The remainder terms on the right-hand side of equations (B-47) and (B-51a) are, respectively, $R_{47} = +\frac{1107}{7560} h^7 y^{ix}$ and $R_{51} = -\frac{64}{7560} h^7 y^{ix}$.

STEP 22.: Predict the values of y_n'', y_n', and y_n for n = 4 as follows:

$$y_n'' = y_{n-6}'' + \frac{3h}{10}(11y_{n-5}''' - 14y_{n-4}''' + 26y_{n-3}''' - 14y_{n-2}''' + 11y_{n-1}''') \qquad (B-48)$$

$$y_n' = y_{n-4}' + \frac{2h}{45}(7y_{n-4}'' + 32y_{n-3}'' + 12y_{n-2}'' + 32y_{n-1}'' + 7y_n'') \qquad (B-49)$$

$$y_n = y_{n-4} + \frac{2h}{45}(7y_{n-4}' + 32y_{n-3}' + 12y_{n-2}' + 32y_{n-1}' + 7y_n') \qquad (B-50)$$

STEP 23.: Evaluate y_n''' from the differential equation, (B-5).

STEP 24.: Correct the values of y_n'', y_n', and y_n for n = 4 as follows:

$$y_n'' = y_{n-4}'' + \frac{2h}{45}(7y_{n-4}''' + 32y_{n-3}''' + 12y_{n-2}''' + 32y_{n-1}''' + 7y_n''') \qquad (B-51)$$

$$y_n' = y_{n-4}' + \frac{2h}{45}(7y_{n-4}'' + 32y_{n-3}'' + 12y_{n-2}'' + 32y_{n-1}'' + 7y_n'') \qquad (B-49)$$

$$y_n = y_{n-4} + \frac{2h}{45}(7y_{n-4}' + 32y_{n-3}' + 12y_{n-2}' + 32y_{n-1}' + 7y_n') \qquad (B-50)$$

STEP 25.: Evaluate y_n''' from the differential equation, (B-5).

STEP 26.: Repeat Steps 24 and 25 until no change, to the desired number of significant figures, occurs in the calculated values.

The remainder terms on the right-hand side of equations (B-48) and (B-51) are, respectively, $R_{48} = +\frac{2214}{7560} h^7 y^{ix}$ and $R_{51} = -\frac{64}{7560} h^7 y^{ix}$, where y^{ix} is evaluated at some point in the interval between x_{n-6} and x_{n-1} for R_{48} and between x_{n-4} and x_n for R_{51}.

STEP 27.: Having completed Steps 22 to 26 inclusive for n = 4, repeat Steps 22 to 26 inclusive for n = 5, 6, 7, ..., until such

time as the solution is cut off.

Details of the conditions under which a solution will be cut off are explained below under "The Program".

If a solution is cut off for being unacceptable, revise the initial estimate of $y''(0)$ and repeat the entire procedure outlined above.

B3. The Program.

The procedure outlined above was programmed in Fortran language and was used with the CDC 1604 Digital Computer. A block diagram of the Milne Method program is given on page 42 and the complete program, designated "Program Milne", is given on pages 46 through 58. The following brief explanation is given as an aid in interpreting the block diagram and the computer program.

For a given value of the parameter β, there is one solution of the equation,

$$y''' + y\,y'' + \beta(1 - y'^2) = 0, \qquad \text{(B-1)}$$

which satisfies the imposed boundary conditions, (B-2), (B-3), and (B-4), as described in section 3, page 4. As has been noted, an essential part of this solution is the value of $y''(0)$. Conversely, for a given value of $y''(0)$, there is one solution and a particular value of β which meets all the specified conditions. As indicated earlier, this latter form of solution is especially of interest for the case of boundary layer separation, for which it is known that $y''(0) = 0$ and the value of β in this case becomes the unknown.

The problem was programmed to provide either of these forms of

solution. That is, if the value of β is specified, the appropriate solution and the value of y"(0) are found. Alternatively, the value of y"(0) may be specified, in which case the appropriate solution and the value of β are found.

The program is written to accept a range of values of the step-width, h, but is designed to output the solution at intervals of $x_{i+1} - x_i = 0.10$. This is accomplished by a loop within a loop (the "N" loop within the "I" loop) and for this reason, the value of h must be chosen such that the value of 0.10/h is an integer.

It will be noted on the block diagram that there are, in sequence, six decisions which test the acceptability of each point calculated. The first three are the imposed boundary conditions, and failure to fulfil any one of these tests will result in the solution being cut off and rejected. Rejection of the solution results in an alteration of the estimate of y"(0), (or β if appropriate), and the commencement of a new solution. It should be noted that if y''' is initially positive, the test y''' : 0 is suppressed until such time as y''' first becomes negative.

The last three decisions apply the convergence criteria. Failure to meet the first convergence test results in the calculation of the next point of the solution, unless an exit is made from the "I" loop at the test I : 101. This latter test checks the current value of x; if I = 101, then x = 10. and the solution is rejected, the assumption being that all solutions will converge before x = 10. Failure to fulfil the last two convergence tests results in rejection of the

solution. If all three convergence tests are fulfilled, the solution is acceptable and complete.

It will be noted that certain items in the block diagram are marked with asterisks. The following notes are applicable.

BLOCK DIAGRAM NOTES:

* No. 1.: The method which is used to alter the value of $y''(0)$ or β, namely by definite increments, requires that:

(i) For a given (fixed) value of β, the initial estimate of $y''(0)$ must be less than the final (correct) value of $y''(0)$.

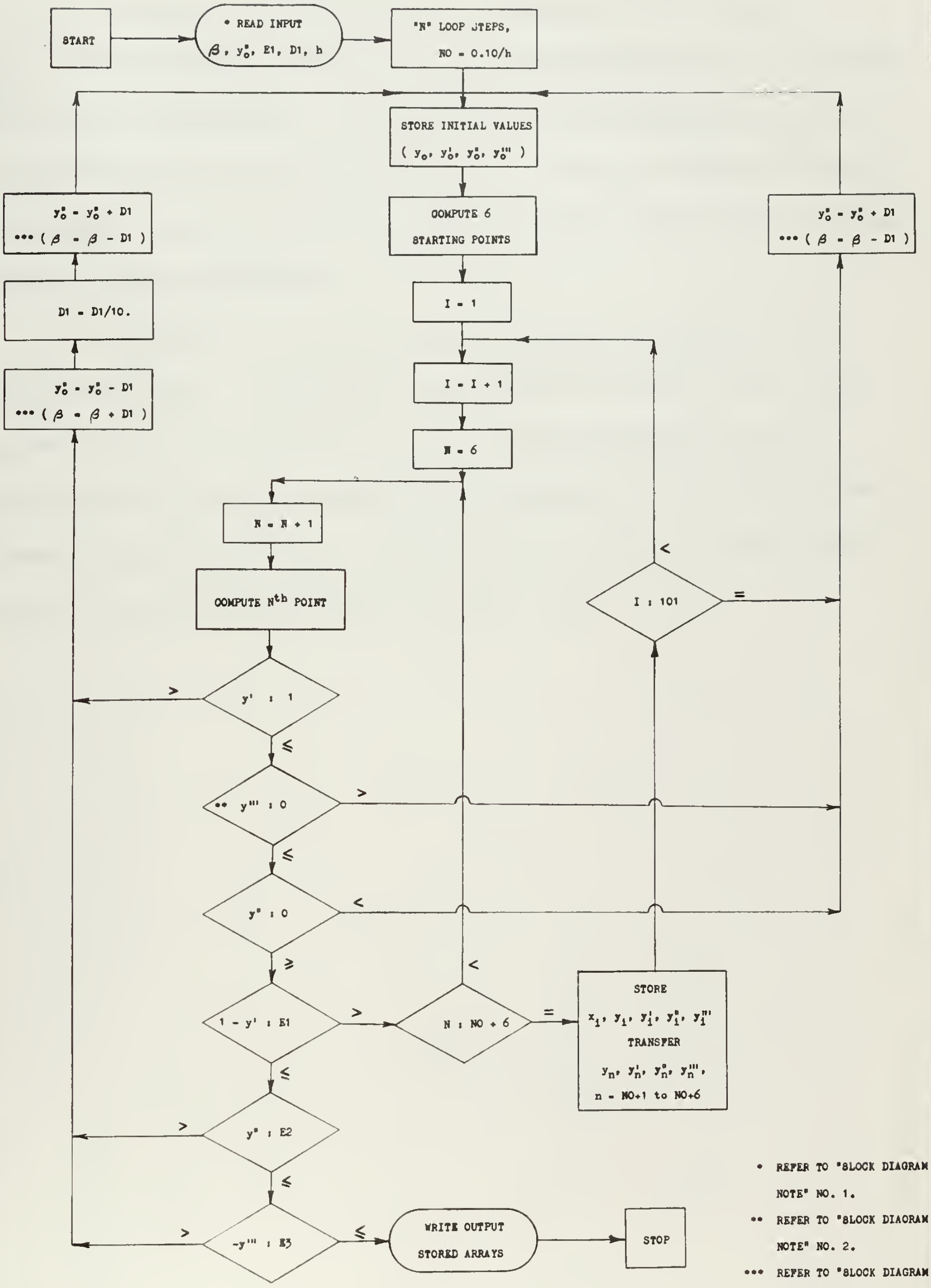
(ii) For a given (fixed) value of $y''(0)$, the initial estimate of β must be greater than the final (correct) value of β.

** No. 2.: If y''' is initially positive, this test is suppressed until such time as y''' first becomes negative.

*** No. 3.: The value of β is altered if it is desired to find the solution for a given (fixed) value of $y''(0)$.

BLOCK DIAGRAM FOR THE MILNE METHOD NUMERICAL SOLUTION

OF THE FALKNER AND SKAN EQUATION



* REFER TO "BLOCK DIAGRAM NOTE" NO. 1.

** REFER TO "BLOCK DIAGRAM NOTE" NO. 2.

*** REFER TO "BLOCK DIAGRAM NOTE" NO. 3.

Figure 4.

To assist the reader in deciphering the program, some of the notation used in the program is explained below.

1. The functions listed at the beginning of the program (e.g. Y3DRVF, THREEF, FOURF, etc.) are compact forms of the equations used in the Milne Method. Where possible, these functions make use of the similarity of a set of equations to reduce the number of function definitions; e.g. the function YCORRF represents the three equations (B-49), (B-50), and (B-51).

2. The variable y is denoted in the program by both the variables W and Y, the variable W being used in the "N" loop, the variable Y being used in the "I" loop. The following list will illustrate this plus the method of subscripting the variables. The illustration is based upon a step-width, h = 0.01, i.e., the number of steps in the "N" loop is NO = 0.10/h = 10.

	x	y			
Starting Points	-.02	W(1)			
	-.01	W(2)			
	0.00	W(3)			Y(1)
	.01	W(4)			
	.02	W(5)			
	.03	W(6)			
1^{st} "N" Loop	.04	W(7)			
	.05	W(8)			
	.06	W(9)			
	.07	W(10)			
	.08	W(11)	W(1)		
	.09	W(12)	W(2)		
	.10	W(13)	W(3)		Y(2)
	.11	W(14)	W(4)		
	.12	W(15)	W(5)		
	.13	W(16)	W(6)		
2^{nd} "N" Loop	.14		W(7)		
	.18		W(11)	W(1)	
	.20		W(13)	W(3)	Y(3)
	.23		W(16)	W(6)	
3^{rd} "N" Loop	.24			W(7)	
	.30			W(13)	Y(4)
	.33			W(16)	

3. The meanings of some of the additional variables are given below.

DW, DY = y'

D2W, D2Y = y''

D3W, D3Y = y'''

D2Y0 = $y''(0)$ = D2Y(1)

D4W3 = $y^{iv}(0)$

PD2W = Predicted value of y''

WC3 = (Second estimate of y'') - (Predicted value of y'')

```
      PROGRAM MILNE
C     FALKNER AND SKAN TWO DIMENSIONAL BOUNDARY LAYER PROBLEM
C     MILNE METHOD NUMERICAL SOLUTION
C                                                JA INGLIS   NHA9   MAR1962
     0DIMENSION W(106),DW(106),D2W(106),D3W(106),PD2W(106),WC3(106),
     1X(101),Y(101),DY(101),D2Y(101),D3Y(101),PD2Y(101),YC3(101)
      COMMON W,DW,D2W,D3W,PD2W,WC3,X,Y,DY,D2Y,D3Y,PD2Y,YC3
      COMMON BETA,D2Y0,H,K,L,NO
      Y3DRVF(A,B,C) = -A*C-BETA*(1.+B)*(1.-B)
      THREEF(A,B,C,D) = H/24.*(A+16.*B+7.*C)+0.25*H*H*D
      FOURF(A,B,C,D) =2./3.*H*(-A-B+5.*C)-2.*H*H*D
      FIVEF(A,B,C) = H/3.*(A+4.*B+C)
      Y2ENDF(A,B,C,D) = H/180.*(53.*A+288.*B-32.*C+3.*D)
      Y2BAKF(P,Q,R,A,B,C,D)=P-4.*H/15.*Q-H*H/3.*R-Y2ENDF(A,B,C,D)
      Y2FWDF(P,Q,R,A,B,C,D)=P+4.*H/15.*Q-H*H/3.*R+Y2ENDF(A,B,C,D)
      Y1ENDF(A,B,C,D)= H/2880.*(-21.*A+944.*B+144.*C-11.*D)
      Y1BAKF(P,Q,R,A,B,C,D)=P-19.*H/30.*Q+11.*H*H/48.*R-Y1ENDF(A,B,C,D)
      Y1FWDF(P,Q,R,A,B,C,D)=P+19.*H/30.*Q+11.*H*H/48.*R+Y1ENDF(A,B,C,D)
     0Y3ESTF(A,B,C,D,P,Q)=3.*H/320.*(-49.*A+496.*B+416.*C-944.*D+
     1   401.*P) + 153.*H*H/16.*Q
      YPREDF(A,B,C,D,P,Q)=A+0.3*H*(11.*(B+Q)-14.*(C+P)+26.*D)
      YCORRF(A,B,C,D,P,Q) = A+2.*H/45.*(7.*(B+Q)+32.*(C+P)+12.*D)
```

```
      3   FORMAT (F10.4)
C         CHOOSE H SUCH THAT VALJE OF 0.10/H IS AN INTEGER
      4   FORMAT (2I5)
      5   FORMAT (2F15.10,2E15.5)
          READ 5, BETA, D2YO, E1, D1
          READ 3, H
          PRINT 5, BETA, D2YO, E1, D1
          PRINT 3, H
          NO = 0.10/H
          NQ = NO + 6
          PRINT 4, NO, NQ
          E2 = E1*1.E+01
          E3 = E1*1.E+02
      6   X(1) = 0.0
          Y(1) = 0.
          DY(1) = 0.
          D2Y(1) = D2YO
          D3Y(1) = -BETA
          PD2Y(1) = D2YO
          YC3(1) = 0.
          W(3) = 0.
          DW(3) = 0.
          D2W(3) = D2YO
```

```
      D3W(3) = -BETA
      D4W3 = 0.
      V1 = -BETA+0.5*H*H*(2.*BETA-1.)*D2Y0*D2Y0
      V2= H*H*H/3.*BETA*(2.-3.*BETA)*D2Y0
      D3W(2) = V1-V2
      D3W(4) = V1+V2
      TEMP 2 =0.
      TEMP4 = 0.
      GO TO 8
    7 TEMP2 = DW(2)
      TEMP4 = DW(4)
    8 D2W(2) = D2W(3) - THREEF(D3W(4),D3W(3),D3W(2),D4W3)
      D2W(4) = D2W(3) + THREEF(D3W(2),D3W(3),D3W(4),D4W3)
      DW(2) =-THREEF(D2W(4),D2W(3),D2W(2),D3W(3))
      DW(4)=THREEF(D2W(2),D2W(3),D2W(4),D3W(3))
      W(2)=-THREEF(DW(4),DW(3),DW(2),D2W(3))
      W(4)=THREEF(DW(2),DW(3),DW(4),D2W(3))
      D3W(2) = Y3DRVF(D2W(2),DW(2),W(2))
      D3W(4) = Y3DRVF(D2W(4),DW(4),W(4))
      IF (ABSF(TEMP2-DW(2))-1.E-8) 10, 10, 7
   10 IF (ABSF(TEMP4-DW(4))-1.E-8) 30, 30, 7
   30 D2W(5)=D2W(3)+FOURF(D3W(2),D3W(3),D3W(4),D4W3)
      DW(5)=FIVEF(D2W(3),D2W(4),D2W(5))
```

```
      W(5)=FIVEF(DW(3),DW(4),DW(5))
      D3W(5)=Y3DRVF(D2W(5),DW(5),W(5))
35    TEMP5 = DW(5)
      D2W(5)=D2W(3)+FIVEF(D3W(3),D3W(4),D3W(5))
      DW(5)=FIVEF(D2W(3),D2W(4),D2W(5))
      W(5)=FIVEF(DW(3),DW(4),DW(5))
      D3W(5)=Y3DRVF(D2W(5),DW(5),W(5))
      IF (ABSF(TEMP5-DW(5))-1.E-8) 40, 40, 35
40    D2W(1)=D2W(3)-FOURF(D3W(4),D3W(3),D3W(2),D4W3)
      DW(1)=-FIVEF(D2W(1),D2W(2),D2W(3))
      W(1)=-FIVEF(DW(1),DW(2),DW(3))
      D3W(1) = Y3DRVF(D2W(1),DW(1),W(1))
45    TEMP1 = DW(1)
      D2W(1)=D2W(3)-FIVEF(D3W(1),D3W(2),D3W(3))
      DW(1)=-FIVEF(D2W(1),D2W(2),D2W(3))
      W(1)=-FIVEF(DW(1),DW(2),DW(3))
      D3W(1) = Y3DRVF(D2W(1),DW(1),W(1))
      IF (ABSF(TEMP1-DW(1))-1.E-8) 50, 50, 45
50    TEMP1 = DW(1)
      TEMP2 = DW(2)
      TEMP4 = DW(4)
      TEMP5 = DW(5)
      D2W(1)=Y2BAKF(D2W(3),D3W(3),D4W3,D3W(1),D3W(2),D3W(4),D3W(5))
      DW(1) =Y2BAKF(DW(3),D2W(3),D3W(3),D2W(1),D2W(2),D2W(4),D2W(5))
```

```
      DW(6)=YCORRF(DW(2),D2W(2),D2W(3),D2W(4),D2W(5),D2W(6))
      W(6)=YCORRF(W(2),DW(2),DW(3),DW(4),DW(5),DW(6))
      D3W(6) = Y3DRVF(D2W(6),DW(6),W(6))
      IF (ABSF(TEMP1-DW(6))-1.E-10) 72, 72, 70
72    ASSIGN 94 TO M3
      DO 150 I = 2, 101
      K = I-1
81    DO 110 N = 7, NQ
      L = N
     0D2W(N)=YPREDF(D2W(N-6),D3W(N-5),D3W(N-4),D3W(N-3),D3W(N-2),
     1          D3W(N-1))
      PD2W(N) = D2W(N)
      DW(N)=YCORRF(DW(N-4),D2W(N-4),D2W(N-3),D2W(N-2),D2W(N-1),D2W(N))
      TEMP1 = DW(N)
      W(N)=YCORRF(W(N-4),DW(N-4),DW(N-3),DW(N-2),DW(N-1),DW(N))
      D3W(N) = Y3DRVF(D2W(N),DW(N),W(N))
     0D2W(N)=YCORRF(D2W(N-4),D3W(N-4),D3W(N-3),D3W(N-2),D3W(N-1),
     1          D3W(N))
      WC3(N) = D2W(N) - PD2W(N)
      GO TO 90
89    TEMP1 = DW(N)
     0D2W(N)=YCORRF(D2W(N-4),D3W(N-4),D3W(N-3),D3W(N-2),D3W(N-1),
     1          D3W(N))
90    DW(N)=YCORRF(DW(N-4),D2W(N-4),D2W(N-3),D2W(N-2),D2W(N-1),D2W(N))
```

```
      W(N)=YCORRF(W(N-4),DW(N-4),DW(N-3),DW(N-2),DW(N-1),DW(N))
      D3W(N) = Y3DRVF(D2W(N),DW(N),W(N))
      IF (ABSF(TEMP1-DW(N))-1.E-10) 92, 92, 89
 92   IF(DW(N)-1.) 93, 93, 320
 93   GO TO M3
 94   IF(D3W(N)) 95, 98, 98
 95   ASSIGN 97 TO M3
 97   IF(D3W(N)) 98, 98, 350
 98   IF(D2W(N)) 350, 102, 102
102   IF((1.-DW(N))-E1)     103, 103, 110
103   IF(D2W(N)-E2)     104, 104, 320
104   IF(-D3W(N)-E3)     500, 500, 320
110   CONTINUE
      DO 120 J = 1, 6
      JNO = J + NO
      W(J) = W(JNO)
      DW(J) = DW(JNO)
      D2W(J) = D2W(JNO)
120   D3W(J) = D3W(JNO)
140   AI = K
      X(I) = 0.1*AI
      Y(I) = W(NO+3)
      DY(I) = DW(NO+3)
```

```
      D2Y(I) = D2W(NO+3)
      D3Y(I) = D3W(NO+3)
      PD2Y(I) = PD2W(NO+3)
      YC3(I) = WC3(NO+3)
150   CONTINUE
      GO TO 350
320   IF(SENSE SWITCH 3) 325, 321
325   PRINT 501, BETA, D2YO, D1
      CALL OUTPUT
321   IF(SENSE SWITCH 1) 323, 322
322   D2YO = D2YO - D1
      D1 = D1*1.E-01
      D2YO = D2YO + D1
      GO TO 6
323   BETA = BETA + D1
      D1 = D1*1.E-01
      BETA = BETA - D1
      GO TO 6
350   IF(SENSE SWITCH 3) 355, 351
355   PRINT 501, BETA, D2YO, D1
      CALL OUTPUT
351   IF(SENSE SWITCH 1) 353, 352
352   D2YO = D2YO + D1
      GO TO 6
```

```
353     BETA = BETA - D1
        GO TO 6
500     PRINT 501, BETA, D2YO, D1
501     FORMAT (2E22.12, E15.5)
        CALL OUTPUT
        PAUSE 500
        IF (SENSE SWITCH 2) 520, 550
520     CALL DELTA
550     STOP 550
        END
        SUBROUTINE OUTPUT
      0 DIMENSION W(106),DW(106),D2W(106),D3W(106),PD2W(106),WC3(106),
      1 X(101),Y(101),DY(101),J2Y(101),D3Y.(101),PD2Y(101),YC3(101)
        COMMON W,DW,D2W,D3W,PD2W,WC3,X,Y,DY,D2Y,D3Y,PD2Y,YC3
        COMMON BETA,D2YO,H,K,L,NO
        IF (SENSE SWITCH 3) 399, 400
398     FORMAT (44H   TIME CUTJFF   COMPUTATION NOT COMPLETED  )
399     WRITE OUTPUT TAPE 4, 398
400   0 WRITE OUTPUT TAPE 4, 401, BETA, D2YO, (X(M),Y(M),DY(M),
      1 D2Y(M),D3Y(M),PD2Y(M),YC3(M),M=1,K)
401   0 FORMAT (50H   FALKNER AND SKAN EQUATION NUMERICAL SOLUTION   ///
      1 12H    BETA   = F12.10//12H   D2F(0) = F12.10///
      2 84H    ETA          F            DF           D2F          D3F
      3     PRED D2F      C3  //(F8.3,F14.8,4F13.8,F10.1))
```

```
 402    FORMAT (F8.3,F14.8,4F13.8,E10.1/1H1)
 403    FORMAT (F8.3,F14.8,4F13.8,E10.1)
 404    IF (L-(NO+3)) 410, 410, 408
 408    ACX = K
        BCX = 0.1*ACX
      0 WRITE OUTPUT TAPE 4,403,BCX,W(NO+3),DW(NO+3),D2W(NO+3),
      1 D3W(NO+3),PD2W(NO+3),WC3(NO+3)
 410    AX = NO*(K-1) + (L-3)
        BX = AX*H
      0 WRITE OUTPUT TAPE 4,402,BX,W(L),DW(L),D2W(L),D3W(L),
      1     PD2W(L),WC3(L)
        IF(SENSE SWITCH 2) 600, 415
 415    RETURN
 600    STOP 600
        END
        SUBROUTINE DELTA
C       TABLE OF DIFFERENCES SUBROUTINE
      0 DIMENSION W(106),DW(106),D2W(106),D3W(106),PD2W(106),WC3(106),
      1 X(101),Y(101),DY(101),D2Y(101),D3Y(101),PD2Y(101),YC3(101)
        COMMON W,DW,D2W,D3W,PD2W,WC3,X,Y,DY,D2Y,D3Y,PD2Y,YC3
        COMMON BETA,D2Y0,H,K,L,NO
```

```
701 0 FORMAT(28H   TABLE OF DIFFERENCES FOR ///12H   BETA   = F12.10//
    1 12H   D2F(0) = F12.10/// 100H   ETA        DF          D1
    2 D2          D3          D4          D5          D6          D7 //
    3 F6.1,F12.8)
702   FORMAT (F6.1,2F12.8)
703   FORMAT (F6.1,3F12.8)
704   FORMAT (F6.1,4F12.8)
705   FORMAT (F6.1,5F12.8)
706   FORMAT (F6.1,6F12.8)
707   FORMAT (F6.1,7F12.8)
708   FORMAT (F6.1,8F12.8)
709   FORMAT (1H1)
      IF (L-(NO+3)) 720, 715, 715
715   PX = K
      X(K+1) = 0.1*PX
      DY(K+1) = DW(NO+3)
      NOT = K+1
      GO TO 725
720   NOT = K
725   WRITE OUTPUT TAPE 4,701,BETA,D2YO,X(1),DY(1)
      AA2 = DY(2) - DY(1)
      WRITE OUTPUT TAPE 4,702,X(2),DY(2),AA2
      AA1 = DY(3) - DY(2)
      BB1 = AA1 - AA2
```

```
WRITE OUTPUT TAPE 4,703,X(3),DY(3),AA1,BB1
AA2 = DY(4) - DY(3)
BB2 = AA2 - AA1
CC2 = BB2 - BB1
WRITE OUTPUT TAPE 4,704,X(4),DY(4),AA2,BB2,CC2
AA1 = DY(5) - DY(4)
BB1 = AA1 - AA2
CC1 = BB1 - BB2
DD1 = CC1 - CC2
WRITE OUTPUT TAPE 4,705,X(5),DY(5),AA1,BB1,CC1,DD1
AA2 = DY(6) -DY(5)
BB2 = AA2 - AA1
CC2 = BB2 - BB1
DD2 = CC2 - CC1
EE2 = DD2 - DD1
WRITE OUTPUT TAPE 4,706,X(6),DY(6),AA2,BB2,CC2,DD2,EE2
AA1 = DY(7) - DY(6)
BB1 = AA1 - AA2
CC1 = BB1 - BB2
DD1 = CC1 - CC2
EE1 = DD1 - DD2
FF1 = EE1 - EE2
WRITE OUTPUT TAPE 4,707,X(7),DY(7),AA1,BB1,CC1,DD1,EE1,FF1
DO 750 M = 8, NOT, 2
```

```
      AA2 = DY(M) - DY(M-1)
      BB2 = AA2 - AA1
      CC2 = BB2 - BB1
      DD2 = CC2 - CC1
      EE2 = DD2 - DD1
      FF2 = EE2 - EE1
      GG2 = FF2 - FF1
      WRITE OUTPUT TAPE 4,708,X(M),DY(M),AA2,BB2,CC2,DD2,EE2,FF2,GG2
      IF (M-NOT) 740, 755, 755
740   AA1 = DY(M+1) - DY(M)
      BB1 = AA1 - AA2
      CC1 = BB1 - BB2
      DD1 = CC1 - CC2
      EE1 = DD1 - DD2
      FF1 = EE1 - EE2
      GG1 = FF1 - FF2
      WRITEOUTPUT TAPE4,708,X(M+1),DY(M+1),AA1,BB1,CC1,DD1,EE1,FF1,GG1
750   CONTINUE
755   WRITE OUTPUT TAPE 4, 709
      IF (SENSE SWITCH 2) 800, 760
760   RETURN
800   STOP 800
      END
      END
```

APPENDIX C

PROGRAMMING THE RUNGE-KUTTA METHOD NUMERICAL SOLUTION OF THE FALKNER AND SKAN EQUATION FOR THE DIGITAL COMPUTER

C1. The Equation.

For the purpose of notation in this section, the variables η and f will be denoted by x and y respectively. The Falkner and Skan equation then becomes:

$$y''' + y\,y'' + \beta(1 - y'^2) = 0, \tag{C-1}$$

where y is a function of x only and the primes denote differentiation with respect to x. The boundary conditions to be imposed are:

$$y = 0,\ y' = 0,\ \text{at } x = 0, \tag{C-2}$$

$$y' \rightarrow 1 \text{ from below as } x \rightarrow \infty, \text{ and} \tag{C-3}$$

$$y'' \rightarrow 0 \text{ from above, } y''' \rightarrow 0 \text{ from below, as } x \rightarrow \infty. \tag{C-4}$$

C2. Description of the Runge-Kutta Method.

The application of the "Runge-Kutta Method" [10] to the numerical solution of equation (C-1) is illustrated below.

Rewrite the differential equation (C-1) as:

$$y_n''' = - y_n\, y_n'' - \beta(1 - y_n'^2) = F(y_n, y_n', y_n'') \tag{C-5}$$

The initial conditions are:

$$y_o = y(0) = 0, \tag{C-6}$$

$$y_o' = y'(0) = 0, \tag{C-7}$$

$$y'' = y''(0) = \text{an ASSUMED value, and} \tag{C-8}$$

β = a SPECIFIED value.

A suitable step-width,

$$h = x_{n+1} - x_n , \tag{C-9}$$

must also be selected. The selection of a suitable value of h has already been discussed on page 5.

Now perform, in sequence, for n = 0, the calculations indicated below.

$$F_1 = F(y_n, y'_n, y''_n) \tag{C-10}$$

$$_1y_n = y_n + \frac{1}{2} h\, y'_n + \frac{1}{8} h^2\, y''_n + \frac{1}{48} h^3\, F_1 \tag{C-11}$$

$$_1y'_n = y'_n + \frac{1}{2} h\, y''_n + \frac{1}{8} h^2\, F_1 \tag{C-12}$$

$$_1y''_n = y''_n + \frac{1}{2} h\, F_1 \tag{C-13}$$

$$F_2 = F({}_1y_n, {}_1y'_n, {}_1y''_n) \tag{C-14}$$

$$_2y''_n = y''_n + \frac{1}{2} h\, F_2 \tag{C-15}$$

$$F_3 = F({}_1y_n, {}_1y'_n, {}_2y''_n) \tag{C-16}$$

$$_3y_n = y_n + h\, y'_n + \frac{1}{2} h^2\, y''_n + \frac{1}{6} h^3\, F_3 \tag{C-17}$$

$$_3y'_n = y'_n + h\, y''_n + \frac{1}{2} h^2\, F_3 \tag{C-18}$$

$$_3y''_n = y''_n + h\, F_3 \tag{C-19}$$

$$F_4 = F({}_3y_n, {}_3y'_n, {}_3y''_n) \tag{C-20}$$

$$y_{n+1} = y_n + h\, y'_n + \frac{1}{2} h^2\, y''_n + \frac{h^3}{20}\left(\frac{3}{2} F_1 + F_2 + F_3 - \frac{1}{6} F_4\right) \tag{C-21}$$

$$y'_{n+1} = y'_n + h\, y''_n + \frac{1}{6} h^2 (F_1 + F_2 + F_3) \tag{C-22}$$

$$y''_{n+1} = y''_n + \frac{1}{6} h(F_1 + 2F_2 + 2F_3 + F_4) \qquad (C-23)$$

$$y'''_{n+1} = F(y_{n+1}, y'_{n+1}, y''_{n+1}) \qquad (C-24)$$

Having completed, for n = 0, the sequence of calculations indicated by equations (C-10) to (C-24) inclusive, repeat the entire procedure for n = 1, 2, 3, ..., until such time as the solution is cut off. Note that equation (C-24) is, in fact, equation (C-10) for the next set of calculations.

If a solution is cut off for being unacceptable, revise the initial estimate of y"(0) and repeat the entire procedure outlined above.

C3. The Program.

The procedure outlined above was programmed in Fortran language and was used with the CDC 1604 Digital Computer. A block diagram of the Runge-Kutta Method program is given on page 63 and the complete program, designated "Program Runge", is given on pages 65 through 71.

The explanation pertaining to "Program Milne" given in Appendix B is equally applicable to "Program Runge" and will not be repeated here. The reader is referred to page 39 for that discussion.

For the sake of clarity, the Block Diagram Notes, which apply to those items marked with asterisks in the block diagram, are repeated below.

BLOCK DIAGRAM NOTES:

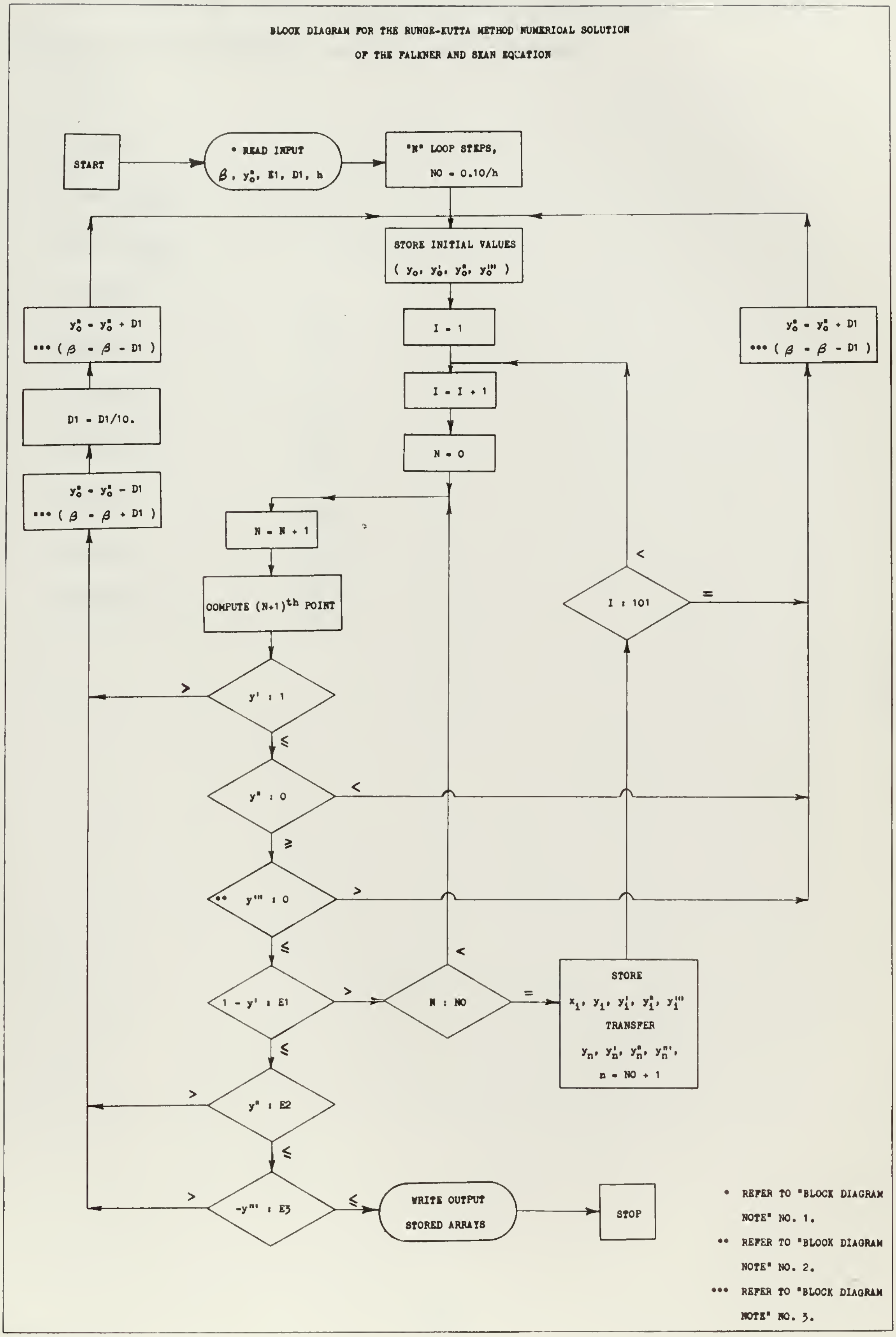
* No. 1.: The method which is used to alter the value of $y''(0)$ or β, namely by definite increments, requires that:

(i) For a given (fixed) value of β, the initial estimate of $y''(0)$ must be less than the final (correct) value of $y''(0)$.

(ii) For a given (fixed) value of $y''(0)$, the initial estimate of β must be greater than the final (correct) value of β.

** No. 2.: If y''' is initially positive, this test is suppressed until such time as y''' first becomes negative.

*** No. 3.: The value of β is altered if it is desired to find the solution for a given (fixed) value of $y''(0)$.



BLOCK DIAGRAM FOR THE RUNGE-KUTTA METHOD NUMERICAL SOLUTION OF THE FALKNER AND SKAN EQUATION

* REFER TO "BLOCK DIAGRAM NOTE" NO. 1.

** REFER TO "BLOCK DIAGRAM NOTE" NO. 2.

*** REFER TO "BLOCK DIAGRAM NOTE" NO. 3.

Figure 5.

To assist the reader in interpreting the program, the meanings of some of the variables used are given below.

W, Y = y

DW, DY = y'

D2W, D2Y = y''

D3W, D3Y = y'''

W1(N) = ${}_1y_n$

DW1(N) = ${}_1y'_n$

D2W1(N) = ${}_1y''_n$

D2W2(N) = ${}_2y''_n$

W3(N) = ${}_3y_n$

DW3(N) = ${}_3y'_n$

D2W3(N) = ${}_3y''_n$

```
      PROGRAM RUNGE
C     FALKNER AND SKAN TWO.DIMENSIONAL BOUNDARY LAYER PROBLEM
C     RUNGE KUTTA METHOD NUMERICAL SOLUTION
C                                        JA INGLIS   NHA9   MAR1962
     0DIMENSION W1(101),DW1(101),D2W1(101),D2W2(101),W3(101),DW3(101),
     1   D2W3(101)
     0DIMENSION W(101),DW(101),D2W(101),D3W(101),X(101),Y(101),
     1   DY(101),D2Y(101),D3Y(101)
      COMMON W,DW,D2W,D3W,X,Y,DY,D2Y,D3Y,BETA,D2YO,H,K,L,NO
      Y3DRVF(A,B,C) = -A*C-BETA*(1.+B)*(1.-B)
    3 FORMAT (F10.4)
C     CHOOSE H SUCH THAT VALUE OF 0.10/H IS AN INTEGER
    4 FORMAT (I5)
    5 FORMAT (2F15.10,2E15.5)
      READ 5, BETA, D2YO, E1, D1
      READ 3, H
      PRINT 5, BETA, D2YO, E1, D1
      PRINT 3, H
      NO = 0.10/H
      PRINT 4, NO
      E2 = E1*1.E+01
      E3 = E1*1.E+02
```

```
    6   X(1) = 0.
        Y(1) = 0.
        DY(1) = 0.
        D2Y(1) = D2YO
        D3Y(1) = -BETA
        W(1) = 0.
        DW(1) = 0.
        D2W(1) = D2YO
        D3W(1) = -BETA
        F1 = D3W(1)
        IF (SENSE SWITCH 2) 25, 20
   20   CONTINUE
        ASSIGN 60 TO M1
        DO 150 I = 2, 101
        K = I-1
        GO TO 30
  301 0 FORMAT(52H   FALKNER AND SKAN EQUATION NUMERICAL SOLUTION    R///
      1 12H   BETA   = F12.10//12H   D2F(0) = F12.10///
      2 59H    ETA           F              DF             D2F            D3F /)
  303   FORMAT (F8.3,F14.8,3F13.8)
   25   WRITE OUTPUT TAPE 4, 301, BETA, D2YO
        WRITE OUTPUT TAPE 4, 303,X(1), Y(1),DY(1),D2Y(1),D3Y(1)
        ASSIGN 112 TO M4
```

```
      IND = 0
      DO 130 J = 2, 301
      MJ = J
30    DO 100 N = 1, NO
      L = N
      W1(N) = W(N) +0.5*H*DW(N)+0.125*H*H*D2W(N)+H*H*H/48.*F1
      DW1(N) = DW(N)+0.5*H*D2W(N)+0.125*H*H*F1
      D2W1(N) = D2W(N) +0.5*H*F1
      F2 = Y3DRVF(W1(N),DW1(N),D2W1(N))
      D2W2(N) = D2W(N) + 0.5*H*F2
      F3 = Y3DRVF(W1(N),DW1(N),D2W2(N))
      W3(N) = W(N) +H*DW(N)+0.5*H*H*D2W(N)+H*H*H/6.*F3
      DW3(N) = DW(N)+H*D2W(N)+0.5*H*H*F3
      D2W3(N) = D2W(N) + H*F3
      F4 = Y3DRVF(W3(N),DW3(N),D2W3(N))
     0 W(N+1)=W(N)+H*DW(N)+0.5*H*H*D2W(N)+H*H*H/20.*(1.5*F1+
     1    F2+F3-F4/6.)
      DW(N+1)=DW(N)+H*D2W(N)+H*H/6.*(F1+F2+F3)
      D2W(N+1)=D2W(N)+H/6.*(F1+F4+2.*(F2+F3))
      D3W(N+1)=Y3DRVF(W(N+1),DW(N+1),D2W(N+1))
      IF (SENSF SWITCH 2) 95, 45
45    IF (DW(N+1)-1.) 50, 50, 320
```

```
 50   IF(D2W(N+1)) 350, 55, 55
 55   GO TO M1
 60   IF (D3W(N+1)) 65, 75, 75
 65   ASSIGN 70 TO M1
 70   IF (D3W(N+1)) 75, 75, 350
 75   IF ((1.-DW(N+1))-E1) 80, 80, 95
 80   IF (D2W(N+1)-E2) 85, 85, 320
 85   IF (-D3W(N+1)-E3) 500, 500, 320
 95   F1 = D3W(N+1)
100   CONTINUE
      W(1) = W(NO+1)
      DW(1) = DW(NO+1)
      D2W(1) = D2W(NO+1)
      D3W(1) = D3W(NO+1)
      IF (SENSE SWITCH 2) 110, 140
110   AJ = MJ-1
      XX = 0.1*AJ
      IF (MJ-121) 115, 115, 111
111   GO TO M4
112   H = 10.*H
      NO = 0.10/H
      ASSIGN 113 TO M4
```

```
113   IND = IND + 1
      IF (IND-10) 129, 114, 114
114   IND = 0
115   WRITE OUTPUT TAPE 4,303,XX,W(NO+1),DW(NO+1),D2W(NO+1),D3W(NO+1)
129   CONTINUE
130   CONTINUE
305   FORMAT (1H1)
      WRITE OUTPUT TAPE 4, 305
      PAUSE 130
      IF (SENSE SWITCH 2) 135, 6
135   STOP 135
140   AI = K
      X(I) = 0.1*AI
      Y(I) = W(NO+1)
      DY(I) = DW(NO+1)
      D2Y(I) = D2W(NO+1)
      D3Y(I) = D3W(NO+1)
150   CONTINUE
      GO TO 350
320   IF(SENSE SWITCH 3) 325, 321
325   PRINT 501, BETA, D2YO, D1
      CALL OUTPUT
```

```
321   IF(SENSE SWITCH 1) 323, 322
322   D2YO = D2YO - D1
      D1 = D1*1.E-01
      D2YO = D2YO + D1
      GO TO 6
323   BETA = BETA + D1
      D1 = D1*1.E-01
      BETA = BETA - D1
      GO TO 6
350   IF(SENSE SWITCH 3) 355, 351
355   PRINT 501, BETA, D2YO, D1
      CALL OUTPUT
351   IF(SENSE SWITCH 1) 353, 352
352   D2YO = D2YO + D1
      GO TO 6
353   BETA = BETA - D1
      GO TO 6
500   PRINT 501, BETA, D2YO, D1
501   FORMAT (2E22.12, E15.5)
      CALL OUTPUT
      PAUSE 500
      IF (SENSE SWITCH 2) 6, 550
550   STOP 550
      END
```

```
      SUBROUTINE OUTPUT
    0 DIMENSION W(101),DW(101),D2W(101),D3W(101),X(101),Y(101),
    1     DY(101),D2Y(101),D3Y(101)
      COMMON W,DW,D2W,D3W,X,Y,DY,D2Y,D3Y,BETA,D2YO,H,K,L,NO
      IF (SENSE SWITCH 3) 399, 400
398   FORMAT (44H  TIME CUTOFF   COMPUTATION NOT COMPLETED   )
399   WRITE OUTPUT TAPE 4, 398
400 0 WRITE OUTPUT TAPE 4, 401, BETA, D2YO, (X(M),Y(M),DY(M),
    1 D2Y(M),D3Y(M),M = 1, K)
401 0 FORMAT(52H   FALKNER AND SKAN EQUATION NUMERICAL SOLUTION    R///
    1 12H   BETA   = F12.10//12H   D2F(0) = F12.10///
    2 59H    ETA           F             DF            D2F           D3F //
    3 (F8.3,F14.8,3F13.8))
402   FORMAT (F8.3,F14.8,3F13.8/1H1)
410   AX = NO*(K-1) + L
      BX = AX*H
      WRITE OUTPUT TAPE 4, 402,BX,W(L+1),DW(L+1),D2W(L+1),D3W(L+1)
      IF(SENSE SWITCH 2) 600, 415
415   RETURN
600   STOP 600
      END
      END
```

APPENDIX D

TABLES OF NUMERICAL SOLUTIONS OF THE FALKNER AND SKAN BOUNDARY LAYER EQUATION

Numerical solutions of the Falkner and Skan boundary layer equation,

$$f''' + f f'' + \beta(1 - f'^2) = 0$$

with the boundary conditions

$$f = 0,\ f' = 0,\ \text{at}\ \eta = 0,\ \text{and}\ f' \to 1\ \text{as}\ \eta \to \infty,$$

are listed in the following tables for various values of the parameter β; f is a function of η and the primes denote differentiation with respect to η. These solutions have been produced by the Milne Method of solution.

Equivalence of the symbols used in the tables:

BETA = β

D2F(0) = $f''(0)$

ETA = η

F = f

DF = f'

D2F = f''

D3F = f'''

```
FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    = -.1988377500

D2F(0)  =  .0000000000

 ETA          F             DF             D2F            D3F

 .000     .00000000     .00000000     .00000000      .19883773
 .100     .00003314     .00099419     .01988376      .19883687
 .200     .00026512     .00397674     .03976700      .19882404
 .300     .00089476     .00894749     .05964715      .19876844
 .400     .00212087     .01590585     .07951757      .19861878
 .500     .00414213     .02485026     .09936541      .19830336
 .600     .00715702     .03577749     .11916968      .19773031
 .700     .01136354     .04868172     .13889919      .19678812
 .800     .01695895     .06355341     .15851067      .19534644
 .900     .02413932     .08037803     .17794673      .19325759
1.000     .03309896     .09913455     .19713520      .19035865
1.100     .04402967     .11979384     .21598583      .18647451
1.200     .05711987     .14231689     .23439116      .18142206
1.300     .07255350     .16665304     .25222510      .17501555
1.400     .09050879     .19273807     .26934307      .16707340
1.500     .11115678     .22049240     .28558292      .15742638
1.600     .13465971     .24981946     .30076657      .14592717
1.700     .16116928     .28060415     .31470275      .13246109
1.800     .19032466     .31271174     .32719081      .11695758
1.900     .22375058     .34598720     .33802579      .09940197
2.000     .26005520     .38025501     .34700447      .07984669
2.100     .29982817     .41531973     .35393269      .05842113
2.200     .34313860     .45096727     .35863337      .03533885
2.300     .39003338     .48696707     .36095519      .01090139
2.400     .44053563     .52307507     .36078133     -.01450280
2.500     .49464355     .55903763     .35803796     -.04040481
2.600     .55232969     .59459617     .35270162     -.06626785
2.700     .61354070     .62949255     .34480519     -.09150590
2.800     .67819771     .66347490     .33444164     -.11550798
2.900     .74619721     .69530371     .32176508     -.13766673
3.000     .81741261     .72775787     .30698890     -.15740959
3.100     .89169636     .75764037     .29038067     -.17423028
3.200     .96888266     .78578335     .27225407     -.18771797
3.300    1.04879052     .81205218     .25295803     -.19758157
3.400    1.13122730     .83634834     .23286386     -.20366696
3.500    1.21599237     .85861100     .21235104     -.20596525
3.600    1.30282091     .87881706     .19179269     -.20461155
3.700    1.39168763     .89697932     .17154165     -.19987409
3.800    1.48221030     .91314635     .15191803     -.19213485
3.900    1.57425290     .92739364     .13319915     -.18186358
4.000    1.66762844     .93982397     .11561217     -.16958763
4.100    1.76216119     .95055963     .09932999     -.15586027
4.200    1.85768842     .95973744     .08447017     -.14123016
4.300    1.95406160     .96750327     .07109676     -.12621428
4.400    2.05114701     .97400638     .05922465     -.11127598
4.500    2.14882588     .97939734     .04882576     -.09680928
4.600    2.24699419     .98381905     .03983647     -.08312961
4.700    2.34556197     .98740861     .03216571     -.07047091
4.800    2.44445240     .99029248     .02570301     -.05898808
4.900    2.54360078     .99258541     .02032608     -.04876376
5.000    2.64295321     .99438964     .01590758     -.03931814
```

(continued on next page)

BETA = -.19883773 (continued)

5.100	2.74246541	.99579465	.01232081	-.03212053
5.200	2.84210141	.99687746	.00944416	-.02560144
5.300	2.94183235	.99770336	.00716443	-.02016430
5.400	3.04163535	.99832681	.00537892	-.01569611
5.500	3.14149247	.99879259	.00399693	-.01207646
5.600	3.24138983	.99913700	.00293944	-.00918483
5.700	3.34131680	.99938906	.00213955	-.00690602
5.800	3.44126535	.99957163	.00154136	-.00513391
5.900	3.54122940	.99970251	.00109905	-.00377370
6.000	3.64120457	.99979539	.00077565	-.00274295
6.100	3.74118756	.99986062	.00054182	-.00197164
6.200	3.84117603	.99990597	.00037463	-.00140161
6.300	3.94116829	.99993717	.00025638	-.00098547
6.400	4.04116314	.99995843	.00017368	-.00068532
6.500	4.14115974	.99997276	.00011645	-.00047141
6.600	4.24115753	.99998232	.00007729	-.00032077
6.700	4.34115610	.99998863	.00005078	-.00021591
6.800	4.44115518	.99999276	.00003302	-.00014377
6.900	4.54115460	.99999544	.00002126	-.00009471
7.000	4.64115424	.99999715	.00001354	-.00006173
7.100	4.74115401	.99999824	.00000854	-.00003981
7.200	4.84115387	.99999892	.00000533	-.00002540
7.300	4.94115379	.99999934	.00000330	-.00001603
7.400	5.04115374	.99999960	.00000202	-.00001001
7.500	5.14115371	.99999976	.00000122	-.00000619
7.600	5.24115369	.99999986	.00000073	-.00000379
7.700	5.34115368	.99999991	.00000044	-.00000229
7.800	5.44115367	.99999995	.00000026	-.00000138
7.900	5.54115366	.99999997	.00000015	-.00000082
8.000	5.64115366	.99999998	.00000009	-.00000048
8.100	5.74115366	.99999999	.00000005	-.00000029
8.200	5.84115366	.99999999	.00000003	-.00000017
8.210	5.85115366	.99999999	.00000002	-.00000014

```
FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    = -.1900000000

D2F(0) =   .0856997000

 ETA           F                 DF                D2F               D3F

 .000       .00000000         .00000000         .08569970         .19000000
 .100       .00046016         .00951992         .10469765         .18993460
 .200       .00196729         .02093902         .12368011         .18967338
 .300       .00471119         .03425461         .14262211         .18910514
 .400       .00888124         .04946088         .16148663         .18810099
 .500       .01466605         .06554768         .18022294         .18651541
 .600       .02225294         .08549901         .19876500         .18418798
 .700       .03182724         .10529146         .21703006         .18094594
 .800       .04357153         .12389246         .23491765         .17660776
 .900       .05766453         .15325847         .25230889         .17098796
1.000       .07428019         .17933313         .26906640         .16390323
1.100       .09358581         .20704547         .28503489         .15517989
1.200       .11574078         .23530810         .30004256         .14466289
1.300       .14089562         .26701576         .31390343         .13222587
1.400       .16918816         .29904401         .32642079         .11778227
1.500       .20074365         .33224938         .33739175         .10129684
1.600       .23567153         .36546403         .34661293         .08279696
1.700       .27406402         .40150605         .35388725         .06238289
1.800       .31599356         .43717045         .35903165         .04023589
1.900       .36151150         .47323599         .36188547         .01662330
2.000       .41064628         .50946683         .36231917       -.00810074
2.100       .46340216         .54561610         .36024275       -.03349968
2.200       .51975833         .58143023         .35561363       -.05906476
2.300       .57966852         .61565405         .34844304       -.08423128
2.400       .64306108         .65103638         .33880060       -.10840067
2.500       .70983969         .68433604         .32691636       -.13096723
2.600       .77988467         .71632785         .31268007       -.15134825
2.700       .85305486         .74580845         .29663727       -.16901521
2.800       .92919007         .77560153         .27898243       -.18352367
2.900      1.00811405         .80256229         .26004911       -.19453935
3.000      1.08963775         .82758074         .24019796       -.20185784
3.100      1.17356297         .85058376         .21980304       -.20541632
3.200      1.25968609         .87153568         .19923748       -.20529583
3.300      1.34780173         .89043753         .17885939       -.20171400
3.400      1.43770641         .90732485         .15899889       -.19500902
3.500      1.52920175         .92226433         .13994725       -.18561617
3.600      1.62209744         .93534946         .12194850       -.17403929
3.700      1.71621365         .94569557         .10519416       -.16081982
3.800      1.81138296         .95543439         .08982100       -.14650590
3.900      1.90745171         .96470862         .07591185       -.13162411
4.000      2.00428082         .97166669         .06349899       -.11665568
4.100      2.10174615         .97745791         .05256959       -.10201849
4.200      2.19973838         .98222839         .04307273       -.08805553
4.300      2.29816246         .98511753         .03492716       -.07502957
4.400      2.39693686         .98725543         .02802939       -.06312369
4.500      2.49579249         .99176107         .02226140       -.05244639
4.600      2.59527157         .99374119         .01749768       -.04304033
4.700      2.69472636         .99528985         .01361132       -.03489315
4.800      2.79431789         .99548858         .01047890       -.02794939
4.900      2.89401474         .99740688         .00798420       -.02212229
5.000      2.99379187         .99810311         .00602078       -.01730482
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(continued on next page)

BETA = -.19 (continued)

5.100	3.09362958	.99862556	.00449350	-.01337930
5.200	3.19351251	.99901357	.00331921	-.01022527
5.300	3.29342887	.99929878	.00242665	-.00772561
5.400	3.39336968	.99950628	.00175593	-.00577095
5.500	3.49332820	.99965570	.00125760	-.00426239
5.600	3.59329940	.99976220	.00089149	-.00311304
5.700	3.69327959	.99983733	.00062551	-.00224839
5.800	3.79326611	.99988979	.00043442	-.00160598
5.900	3.89325701	.99992605	.00029863	-.00113454
6.000	3.99325094	.99995086	.00020320	-.00079274
6.100	4.09324692	.99996766	.00013686	-.00054790
6.200	4.19324429	.99997892	.00009124	-.00037458
6.300	4.29324258	.99998639	.00006021	-.00025333
6.400	4.39324148	.99999130	.00003933	-.00016949
6.500	4.49324078	.99999449	.00002543	-.00011218
6.600	4.59324034	.99999654	.00001628	-.00007346
6.700	4.69324006	.99999785	.00001031	-.00004759
6.800	4.79323987	.99999867	.00000647	-.00003051
6.900	4.89323979	.99999919	.00000402	-.00001935
7.000	4.99323972	.99999951	.00000247	-.00001214
7.100	5.09323968	.99999970	.00000150	-.00000754
7.200	5.19323966	.99999982	.00000091	-.00000463
7.300	5.29323965	.99999989	.00000054	-.00000282
7.400	5.39323964	.99999993	.00000032	-.00000170
7.500	5.49323963	.99999996	.00000019	-.00000101
7.600	5.59323963	.99999997	.00000011	-.00000060
7.700	5.69323963	.99999998	.00000006	-.00000036
7.800	5.79323962	.99999998	.00000004	-.00000021
7.900	5.89323962	.99999999	.00000002	-.00000013
8.000	5.99323962	.99999999	.00000001	-.00000008
8.100	6.09323962	.99999999	.00000001	-.00000006
8.110	6.10323962	.99999999	.00000000	-.00000002

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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA   = -.1800000000

D2F(0) =  .1286362000

 ETA          F             DF            D2F           D3F

 .000      .00000000     .00000000     .12863620     .18000000
 .100      .00067318     .01376352     .14663195     .17986719
 .200      .00281265     .02932541     .16459792     .17938225
 .300      .00659805     .04568072     .18249196     .17940367
 .400      .01220825     .06581952     .20025701     .17677541
 .500      .01982086     .08572540     .21781977     .17432879
 .600      .02961143     .10937373     .23508955     .17088539
 .700      .04175255     .13372993     .25195749     .16626107
 .800      .05641281     .15974761     .26829624     .16027118
 .900      .07375547     .18736671     .28396024     .15273725
1.000      .09393704     .21551176     .29878672     .14349494
1.100      .11710562     .24709021     .31259755     .13240343
1.200      .14339913     .27899107     .32520215     .11935580
1.300      .17294359     .31208380     .33640138     .10429020
1.400      .20585067     .34521774     .34599266     .08720117
1.500      .24221617     .38122206     .35377613     .06815025
1.600      .28211777     .41590636     .35956182     .04727526
1.700      .32561317     .45306207     .36317764     .02479683
1.800      .37273842     .48946466     .36447786     .00102148
1.900      .42350645     .52587671     .36335164    -.02366009
2.000      .47770587     .56205179     .35973116    -.04878003
2.100      .53590053     .59773915     .35359871    -.07380631
2.200      .59742911     .63268900     .34499224    -.09816157
2.300      .66240563     .66565829     .33400572    -.12124725
2.400      .73072038     .69941667     .32080487    -.14247172
2.500      .80224151     .73075234     .30559497    -.16128078
2.600      .87631714     .76047763     .28864555    -.17718808
2.700      .95427803     .78843378     .27026723    -.18980308
2.800     1.03444069     .81449488     .25080388    -.19885408
2.900     1.11711078     .83857053     .23061993    -.20420411
3.000     1.20208679     .86060722     .21008642    -.20585817
3.100     1.28916367     .88058830     .18956681    -.20396109
3.200     1.37813652     .89853250     .16940358    -.19878618
3.300     1.46880396     .91449126     .14990619    -.19071577
3.400     1.56097124     .92854439     .13134148    -.18021548
3.500     1.65445289     .94079795     .11392680    -.16780467
3.600     1.74907491     .95137411     .09782615    -.15402555
3.700     1.84467639     .96041070     .08314941    -.13941372
3.800     1.94111059     .96805343     .06995434    -.12447205
3.900     2.03824558     .97445134     .05825095    -.10964971
4.000     2.13596430     .97975233     .04800765    -.09532726
4.100     2.23416426     .98409937     .03915863    -.08180809
4.200     2.33275689     .98762748     .03161171    -.06931589
4.300     2.43166663     .99046144     .02525625    -.05799728
4.400     2.53082982     .99271439     .01997052    -.04792872
4.500     2.63019351     .99448700     .01562825    -.03912612
4.600     2.72971416     .99586731     .01210416    -.03155621
4.700     2.82935643     .99593109     .00927825    -.02514836
4.800     2.92909198     .99774250     .00703896    -.01980598
4.900     3.02849832     .99835506     .00528527    -.01541685
5.000     3.12875784     .99881275     .00392778    -.01186192
5.100     3.22865690     .99715123     .00288905    -.00902231
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(continued on next page)

BETA = -.18 (continued)

5.200	3.32858506	.99939898	.00210327	-.00678460
5.300	3.42853443	.99957847	.00151556	-.00504442
5.400	3.52849907	.99970718	.00108092	-.00370862
5.500	3.62847462	.99979854	.00076306	-.00269624
5.600	3.72845787	.99986272	.00053312	-.00193857
5.700	3.82844651	.99990735	.00036878	-.00137849
5.800	3.92843888	.99993808	.00025247	-.00096951
5.900	4.02843380	.99995901	.00017109	-.00067445
6.000	4.12843046	.99997312	.00011476	-.00046411
6.100	4.22842827	.99998255	.00007620	-.00031592
6.200	4.32842686	.99998878	.00005008	-.00021273
6.300	4.42842596	.99999285	.00003258	-.00014172
6.400	4.52842538	.99999549	.00002098	-.00009340
6.500	4.62842502	.99999718	.00001338	-.00006090
6.600	4.72842480	.99999825	.00000844	-.00003929
6.700	4.82842466	.99999893	.00000527	-.00002508
6.800	4.92842458	.99999935	.00000326	-.00001584
6.900	5.02842453	.99999961	.00000200	-.00000989
7.000	5.12842449	.99999976	.00000121	-.00000612
7.100	5.22842448	.99999986	.00000072	-.00000374
7.200	5.32842447	.99999991	.00000043	-.00000226
7.300	5.42842446	.99999995	.00000025	-.00000135
7.400	5.52842445	.99999997	.00000015	-.00000079
7.500	5.62842445	.99999998	.00000008	-.00000046
7.600	5.72842445	.99999999	.00000005	-.00000026
7.700	5.82842445	.99999999	.00000002	-.00000014
7.720	5.84842445	.99999999	.00000002	-.00000012

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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    = -.1600000000

D2F(0) =   .1907798400

 ETA           F              DF             D2F            D3F

 .000      .00000000      .00000000      .19077984      .16000000
 .100      .00098056      .01987777      .20677119      .15973403
 .200      .00402879      .04135235      .22270536      .15882916
 .300      .00930396      .06441455      .23851012      .15711703
 .400      .01696405      .08904710      .25409597      .15442080
 .500      .02716484      .11522288      .26935543      .15055879
 .600      .04005879      .14290313      .28416281      .14534937
 .700      .05579389      .17203561      .29837452      .13861713
 .800      .07451209      .20255285      .31183006      .13020046
 .900      .09634783      .23437058      .32435380      .11996049
1.000      .12142616      .26738630      .33575779      .10779095
1.100      .14986098      .30147826      .34584552      .09362899
1.200      .18175299      .33650485      .35441682      .07746600
1.300      .21718774      .37230447      .36127381      .05935806
1.400      .25623364      .40869616      .36622783      .03943490
1.500      .29894009      .44548085      .36910719      .01790655
1.600      .34533576      .48244352      .36976533     -.00493347
1.700      .39542714      .51935607      .36808921     -.02870938
1.800      .44919740      .55598111      .36400724     -.05296950
1.900      .50660570      .59207649      .35749630     -.07719839
2.000      .56758697      .62740037      .34858729     -.10083460
2.100      .63205218      .66171689      .33736862     -.12329365
2.200      .69988928      .69480194      .32398718     -.14399511
2.300      .77096461      .72644896      .30864649     -.16239202
2.400      .84512499      .75647439      .29160183     -.17800055
2.500      .92220021      .78472247      .27315255     -.19042764
2.600     1.00200606      .81106916      .25363181     -.19939392
2.700     1.08434764      .83542493      .23339426     -.20475009
2.800     1.16902286      .85773622      .21280265     -.20648499
2.900     1.25582611      .87798558      .19221391     -.20472464
3.000     1.34455179      .89619039      .17196591     -.19972223
3.100     1.43499766      .91240039      .15236555     -.19184013
3.200     1.52696796      .92669402      .13367904     -.18152547
3.300     1.62027599      .93917400      .11612478     -.16928185
3.400     1.71474636      .94996233      .09986930     -.15563907
3.500     1.81021659      .95919499      .08502600     -.14112429
3.600     1.90653832      .96701672      .07165684     -.12623592
3.700     2.00357785      .97357602      .05977625     -.11142242
3.800     2.10121637      .97902072      .04935701     -.09706681
3.900     2.19934963      .98349410      .04033726     -.08347744
4.000     2.29788735      .98713168      .03262822     -.07088466
4.100     2.39675236      .99005935      .02612189     -.05944267
4.200     2.49587949      .99239236      .02069845     -.04923565
4.300     2.59521440      .99423147      .01623276     -.04028689
4.400     2.69471233      .99566667      .01259998     -.03256965
4.500     2.79433686      .99677521      .00967993     -.02601871
4.600     2.89405868      .99762266      .00736042     -.02054165
4.700     2.99385453      .99826390      .00553943     -.01602919
4.800     3.09370611      .99874413      .00412634     -.01236405
4.900     3.19359922      .99910012      .00304232     -.00942813
5.000     3.29352298      .99936132      .00222020     -.00710796
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BETA = -.16 (continued)

5.100	3.39346911	.99955101	.00160372	-.00529852
5.200	3.49343140	.99968736	.00114662	-.00390562
5.300	3.59340527	.99978439	.00081147	-.00284694
5.400	3.69338733	.99985272	.00056844	-.00205235
5.500	3.79337513	.99990036	.00039416	-.00146331
5.600	3.89336691	.99993324	.00027054	-.00103193
5.700	3.99336143	.99995569	.00018380	-.00071982
5.800	4.09335781	.99997088	.00012361	-.00049668
5.900	4.19335544	.99998104	.00008229	-.00033902
6.000	4.29335390	.99998778	.00005423	-.00022892
6.100	4.39335292	.99999219	.00003538	-.00015292
6.200	4.49335229	.99999506	.00002284	-.00010107
6.300	4.59335190	.99999691	.00001460	-.00006608
6.400	4.69335165	.99999808	.00000924	-.00004275
6.500	4.79335150	.99999882	.00000579	-.00002737
6.600	4.89335140	.99999928	.00000359	-.00001733
6.700	4.99335135	.99999956	.00000220	-.00001086
6.800	5.09335131	.99999974	.00000134	-.00000674
6.900	5.19335129	.99999984	.00000081	-.00000413
7.000	5.29335128	.99999990	.00000048	-.00000251
7.100	5.39335127	.99999994	.00000028	-.00000151
7.200	5.49335127	.99999996	.00000017	-.00000090
7.300	5.59335126	.99999998	.00000010	-.00000053
7.400	5.69335126	.99999998	.00000006	-.00000031
7.500	5.79335126	.99999999	.00000003	-.00000018
7.600	5.89335126	.99999999	.00000002	-.00000010

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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    = -.1400000000

D2F(0)  =  .2397359400

 ETA          F              DF             D2F            D3F

 .000      .00000000      .00000000      .23973594      .14000000
 .100      .00122201      .02467327      .25372299      .13960472
 .200      .00498117      .05074184      .26762679      .13830644
 .300      .01141646      .07819258      .28134841      .13593203
 .400      .02066498      .10700159      .29477146      .13230565
 .500      .03286086      .13713247      .30776202      .12725393
 .600      .04813387      .16853457      .32016924      .12061247
 .700      .06660795      .20114134      .33182658      .11223362
 .800      .08839952      .23486890      .34255395      .10199552
 .900      .11361569      .26961479      .35216078      .08981211
1.000      .14235238      .30525714      .36045014      .07564360
1.100      .17469230      .34165429      .36722383      .05950695
1.200      .21070304      .37864493      .37228862      .04148558
1.300      .25043509      .41604898      .37546329      .02173727
1.400      .29392006      .45366917      .37658634      .00049952
1.500      .34116907      .49129337      .37552408     -.02190889
1.600      .39217141      .52869783      .37217869     -.04509084
1.700      .44689360      .56565113      .36649564     -.06857912
1.800      .50527878      .60191882      .35847007     -.09185020
1.900      .56724676      .63726865      .34815152     -.11434340
2.000      .63269442      .67147617      .33564646     -.13548488
2.100      .70149686      .70433044      .32111830     -.15471487
2.200      .77350898      .73563959      .30478450     -.17151673
2.300      .84856766      .76523597      .28691084     -.18544531
2.400      .92649441      .79298058      .26780295     -.19615249
2.500     1.00709845      .81376656      .24779556     -.20340754
2.600     1.09017998      .84252142      .22724000     -.20711043
2.700     1.17553375      .86420812      .20649084     -.20729675
2.800     1.26295256      .88382464      .18589243     -.20413376
2.900     1.35223069      .90140238      .16576629     -.19790795
3.000     1.44316711      .91700328      .14640015     -.18900519
3.100     1.53556838      .93071597      .12803921     -.17788547
3.200     1.62925104      .94265124      .11088013     -.16505435
3.300     1.72404363      .95293696      .09506797     -.15103377
3.400     1.81978810      .96171290      .08069596     -.13633438
3.500     1.91634076      .96912566      .06780782     -.12143152
3.600     2.01357275      .97532392      .05640233     -.10674614
3.700     2.11136996      .98045425      .04643948     -.09263159
3.800     2.20963271      .98465755      .03784761     -.07936639
3.900     2.30827499      .98806630      .03053107     -.06715261
4.000     2.40722356      .99080254      .02437768     -.05611907
4.100     2.50641678      .99297655      .01926576     -.04632838
4.200     2.60580341      .99468624      .01507033     -.03778641
4.300     2.70534141      .99601706      .01166815     -.03045332
4.400     2.80499665      .99704239      .00894182     -.02425486
4.500     2.90474178      .99782431      .00678260     -.01909318
4.600     3.00455513      .99841453      .00509234     -.01485662
4.700     3.10441972      .99885551      .00378435     -.01142794
4.800     3.20432241      .99918163      .00278370     -.00869083
4.900     3.30425314      .99942036      .00202682     -.00653488
5.000     3.40420430      .99959334      .00146074     -.00485883
```

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BETA = -.14 (continued)

5.100	3.50417019	.99971741	.00104208	-.00357251
5.200	3.60414659	.99980550	.00073587	-.00259773
5.300	3.70413042	.99986741	.00051437	-.00186818
5.400	3.80411944	.99991047	.00035590	-.00132884
5.500	3.90411207	.99994013	.00024377	-.00093493
5.600	4.00410715	.99996034	.00016527	-.00065067
5.700	4.10410392	.99997398	.00011092	-.00044795
5.800	4.20410180	.99998310	.00007369	-.00030508
5.900	4.30410043	.99998912	.00004847	-.00020555
6.000	4.40409956	.99999306	.00003155	-.00013702
6.100	4.50409900	.99999562	.00002033	-.00009036
6.200	4.60409865	.99999726	.00001297	-.00005896
6.300	4.70409843	.99999830	.00000819	-.00003807
6.400	4.80409830	.99999895	.00000512	-.00002432
6.500	4.90409821	.99999936	.00000317	-.00001537
6.600	5.00409816	.99999961	.00000194	-.00000961
6.700	5.10409813	.99999977	.00000118	-.00000594
6.800	5.20409811	.99999986	.00000071	-.00000364
6.900	5.30409810	.99999991	.00000042	-.00000220
7.000	5.40409810	.99999995	.00000025	-.00000132
7.100	5.50409809	.99999997	.00000014	-.00000077
7.200	5.60409809	.99999998	.00000008	-.00000045
7.300	5.70409809	.99999998	.00000004	-.00000025
7.400	5.80409809	.99999999	.00000002	-.00000013
7.500	5.90409808	.99999999	.00000001	-.00000006
7.520	5.92409808	.99999999	.00000001	-.00000005

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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    = -.1000000000

D2F(0) =   .3192697500

 ETA          F              DF             D2F            D3F

 .000     .00000000      .00000000      .31926975      .10000000
 .100     .00161301      .03242645      .32924875      .09936377
 .200     .00651839      .06584540      .33909679      .09735607
 .300     .01481451      .10023664      .34866934      .09382990
 .400     .02659697      .13556480      .35780748      .08864562
 .500     .04195699      .17177793      .36633902      .08167875
 .600     .06097975      .20380628      .37408042      .07282866
 .700     .08374249      .24556128      .38083969      .06202829
 .800     .11031265      .28493493      .38642023      .04925417
 .900     .14074586      .32379949      .39062567      .03453644
1.000     .17508403      .36300787      .39326575      .01796798
1.100     .21335340      .40239452      .39416293     -.00028813
1.200     .25556281      .44177707      .39315972     -.01999370
1.300     .30170213      .48095875      .39012628     -.04083406
1.400     .35174094      .51973145      .38496794     -.06242106
1.500     .40562761      .55787970      .37763224     -.08430104
1.600     .46328878      .59518512      .36811480     -.10596799
1.700     .52462932      .63143148      .35646373     -.12688190
1.800     .58953280      .66541011      .34278192     -.14649143
1.900     .65786254      .69992536      .32722686     -.16425985
2.000     .72946316      .73179997      .31000788     -.17969245
2.100     .80416263      .76187995      .29138053     -.19236346
2.200     .88177508      .79003889      .27163846     -.20194017
2.300     .96210320      .81618119      .25110312     -.20820229
2.400    1.04494198      .84024431      .23011181     -.21105454
2.500    1.13008177      .86219970      .20900487     -.21053142
2.600    1.21731179      .88205247      .18811281     -.20679360
2.700    1.30642338      .89983975      .16774424     -.20011616
2.800    1.39721308      .91562802      .14817529     -.19086991
2.900    1.48948540      .92950936      .12964118     -.17949742
3.000    1.58305515      .94159709      .11233046     -.16648582
3.100    1.67774934      .95202091      .09638185     -.15233897
3.200    1.77340856      .96092187      .08188400     -.13755087
3.300    1.86988787      .96844747      .06887763     -.12258249
3.400    1.96705719      .97474706      .05735980     -.10784318
3.500    2.06480132      .97996773      .04728978     -.09367768
3.600    2.16301950      .98425091      .03859596     -.08035879
3.700    2.26162470      .98772964      .03118309     -.06808544
3.800    2.36054271      .99052652      .02493968     -.05698547
3.900    2.45971099      .99275252      .01974472     -.04712206
4.000    2.55907748      .99450627      .01547375     -.03850279
4.100    2.65859939      .99587397      .01200391     -.03109007
4.200    2.75824190      .99692983      .00921788     -.02481207
4.300    2.85797706      .99773670      .00700684     -.01957323
4.400    2.95778270      .99834707      .00527225     -.01526386
4.500    3.05764138      .99880412      .00392694     -.01176815
4.600    3.15753959      .99914290      .00289535     -.00897082
4.700    3.25746696      .99939149      .00211318     -.00676193
4.800    3.35741562      .99957205      .00152673     -.00504030
4.900    3.45737968      .99970188      .00109190     -.00371550
5.000    3.55735475      .99979429      .00077304     -.00270883
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BETA = -.10 (continued)

5.100	3.65733763	.99985940	.00054177	-.00195333
5.200	3.75732598	.99990482	.00037587	-.00139323
5.300	3.85731813	.99993617	.00025814	-.00098298
5.400	3.95731288	.99995761	.00017551	-.00068605
5.500	4.05730942	.99997211	.00011812	-.00047368
5.600	4.15730715	.99998183	.00007870	-.00032355
5.700	4.25730568	.99998827	.00005191	-.00021865
5.800	4.35730473	.99999250	.00003389	-.00014619
5.900	4.45730413	.99999525	.00002191	-.00009671
6.000	4.55730375	.99999702	.00001402	-.00006330
6.100	4.65730351	.99999815	.00000888	-.00004099
6.200	4.75730336	.99999886	.00000557	-.00002627
6.300	4.85730327	.99999930	.00000346	-.00001666
6.400	4.95730322	.99999958	.00000213	-.00001046
6.500	5.05730318	.99999974	.00000129	-.00000650
6.600	5.15730316	.99999984	.00000078	-.00000400
6.700	5.25730315	.99999991	.00000047	-.00000243
6.800	5.35730314	.99999994	.00000028	-.00000147
6.900	5.45730314	.99999996	.00000016	-.00000088
7.000	5.55730314	.99999997	.00000010	-.00000053
7.100	5.65730314	.99999998	.00000006	-.00000032
7.200	5.75730313	.99999999	.00000004	-.00000020
7.270	5.82730313	.99999999	.00000001	-.00000008

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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    =   .0000000000

D2F(0)  =   .4695999900

 ETA          F             DF            D2F           D3F

 .000     .00000000     .00000000     .46959999    -.00000000
 .100     .00234798     .04695908     .46956324    -.00110253
 .200     .00939141     .09390530     .46930606    -.00440745
 .300     .02112754     .14080562     .46860878    -.00990055
 .400     .03754920     .18760514     .46725420    -.01754502
 .500     .05864268     .23422747     .46503036    -.02727063
 .600     .08438557     .28057546     .46173443    -.03896372
 .700     .11474475     .32653231     .45717749    -.05245872
 .800     .14967452     .37196325     .45119002    -.06753165
 .900     .18911487     .41671779     .44362802    -.08389666
1.000     .23299010     .46063258     .43437915    -.10120604
1.100     .28120764     .50353486     .42336879    -.11905454
1.200     .33365738     .54524651     .41056536    -.13698816
1.300     .39021129     .58558863     .39598460    -.15451766
1.400     .45072362     .62438632     .37969220    -.17113624
1.500     .51503153     .66147384     .36180452    -.18634074
1.600     .58275630     .69569952     .34248711    -.19965502
1.700     .65430492     .72993058     .32195078    -.21065398
1.800     .72887224     .76105734     .30044531    -.21898625
1.900     .80644344     .78999667     .27825114    -.22439381
2.000     .88679683     .81569463     .25566917    -.22672661
2.100     .96970685     .84112792     .23300956    -.22595097
2.200    1.05494715     .86330422     .21057993    -.22215069
2.300    1.14229369     .88326138     .18867385    -.21552095
2.400    1.23152763     .90106545     .16756036    -.20635521
2.500    1.32243803     .91580777     .14747497    -.19502651
2.600    1.41482420     .93060127     .12861287    -.18196459
2.700    1.50849765     .94257618     .11112445    -.16763097
2.800    1.60328357     .95287546     .09511344    -.15249381
2.900    1.69902191     .96165008     .08063738    -.13700467
3.000    1.79556792     .96905461     .06771034    -.12157852
3.100    1.89279230     .97524299     .05630741    -.10657823
3.200    1.99058098     .98036499     .04637042    -.09230407
3.300    2.08883451     .98456314     .03781457    -.07898838
3.400    2.18746726     .98797047     .03053526    -.06679489
3.500    2.28640632     .99070883     .02441483    -.05582222
3.600    2.38559040     .99288791     .01932873    -.04611044
3.700    2.48496851     .99460485     .01515100    -.03764975
3.800    2.58449880     .99594428     .01175871    -.03039036
3.900    2.68414722     .99597888     .00903551    -.02425264
4.000    2.78388647     .99777010     .00687411    -.01913673
4.100    2.88369485     .99336919     .00517781    -.01493124
4.200    2.98355532     .99881831     .00386137    -.01152060
4.300    3.08345466     .99915166     .00285100    -.00879094
4.400    3.18338271     .99939662     .00208408    -.00663443
4.500    3.28333176     .99957484     .00150832    -.00495230
4.600    3.38329602     .99970321     .00108076    -.00365652
4.700    3.48327118     .99979476     .00076670    -.00267061
4.800    3.58325408     .99985940     .00053849    -.00192953
4.900    3.68324241     .99990459     .00037444    -.00137916
5.000    3.78323453     .99993587     .00025778    -.00097525
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BETA = 0.00 (continued)

5.100	3.88322926	.99995730	.00017570	-.00068229
5.200	3.98322577	.99997184	.00011857	-.00047227
5.300	4.08322347	.99998160	.00007921	-.00032345
5.400	4.18322198	.99998810	.00005240	-.00021918
5.500	4.28322102	.99999237	.00003431	-.00014697
5.600	4.38322040	.99999516	.00002225	-.00009751
5.700	4.48322001	.99999696	.00001428	-.00006402
5.800	4.58321977	.99999811	.00000908	-.00004159
5.900	4.68321962	.99999883	.00000571	-.00002674
6.000	4.78321953	.99999929	.00000356	-.00001701
6.100	4.88321947	.99999957	.00000219	-.00001071
6.200	4.98321944	.99999974	.00000134	-.00000668
6.300	5.08321942	.99999985	.00000081	-.00000412
6.400	5.18321941	.99999991	.00000049	-.00000251
6.500	5.28321940	.99999995	.00000029	-.00000152
6.600	5.38321940	.99999997	.00000017	-.00000091
6.700	5.48321940	.99999998	.00000010	-.00000054
6.770	5.55321939	.99999999	.00000007	-.00000036

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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    =   .1000000000

D2F(0) =   .5870352200

 ETA            F              DF              D2F             D3F

 .000       .00000000       .00000000       .58703522      -.10000000
 .100       .00291849       .05320239       .57699010      -.10134519
 .200       .01160665       .11538920       .56668099      -.10524580
 .300       .02596121       .17152159       .55586283      -.11148890
 .400       .04587377       .22653735       .54431298      -.11983777
 .500       .07122870       .28035319       .53183405      -.13002206
 .600       .10190105       .33286754       .51825770      -.14173092
 .700       .13775494       .38396360       .50344882      -.15460976
 .800       .17864222       .43351292       .48730996      -.16826079
 .900       .22440143       .48137936       .46978543      -.18224792
1.000       .27485734       .52742342       .45086467      -.19610592
1.100       .32982076       .57150692       .43058458      -.20935372
1.200       .38908896       .61349781       .40903029      -.22151121
1.300       .45244651       .65327487       .38633405      -.23211869
1.400       .51966659       .69073239       .36267224      -.24075753
1.500       .59051277       .72578427       .33826016      -.24707067
1.600       .66474113       .75836762       .31334497      -.25078115
1.700       .74210275       .78844549       .28819687      -.25170706
1.800       .82234638       .81600867       .26309906      -.24977154
1.900       .90522128       .84107650       .23833699      -.24500674
2.000       .99048005       .86369651       .21418751      -.23755129
2.100      1.07788143       .88394305       .19090864      -.22764135
2.200      1.16719282       .90191497       .16873049      -.21559596
2.300      1.25819259       .91773238       .14784792      -.20179788
2.400      1.35067205       .93153293       .12841521      -.18667148
2.500      1.44443697       .94346750       .11054314      -.17065950
2.600      1.53930867       .95369584       .09429819      -.15420044
2.700      1.63512474       .96238221       .07970411      -.13770820
2.800      1.73173921       .96969121       .06674518      -.12155514
2.900      1.82902245       .97578410       .05537109      -.10605951
3.000      1.92686066       .98081564       .04550275      -.09147753
3.100      2.02515507       .98493148       .03703870      -.07800010
3.200      2.12382093       .98826629       .02986163      -.06575373
3.300      2.22278639       .99094248       .02384464      -.05480485
3.400      2.32199114       .99306953       .01885691      -.04516688
3.500      2.42138521       .99474384       .01476856      -.03680885
3.600      2.52092762       .99604905       .01145464      -.02966496
3.700      2.62058512       .99705667       .00879812      -.02364403
3.800      2.72033105       .99782700       .00669198      -.01863853
3.900      2.82014429       .99841020       .00504043      -.01453246
4.000      2.92000824       .99884743       .00375944      -.01120798
4.100      3.01991003       .99917202       .00277661      -.00855064
4.200      3.11983978       .99941064       .00203066      -.00645318
4.300      3.21979000       .99958434       .00147057      -.00481806
4.400      3.31975504       .99970954       .00105453      -.00355887
4.500      3.41973071       .99979891       .00074878      -.00260082
4.600      3.51971395       .99986207       .00052645      -.00188055
4.700      3.61970250       .99990628       .00036651      -.00134539
4.800      3.71969475       .99993691       .00025265      -.00095239
4.900      3.81968956       .99995792       .00017245      -.00066711
5.000      3.91968611       .99997220       .00011655      -.00046239
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BETA = .10 (continued)

5.100	4.01968385	.99998181	.00007799	-.00031714
5.200	4.11968237	.99998821	.00005168	-.00021525
5.300	4.21968142	.99999243	.00003390	-.00014458
5.400	4.31968081	.99999518	.00002202	-.00009610
5.500	4.41968042	.99999697	.00001417	-.00006321
5.600	4.51968018	.99999811	.00000902	-.00004115
5.700	4.61968003	.99999883	.00000569	-.00002651
5.800	4.71967993	.99999928	.00000355	-.00001690
5.900	4.81967988	.99999957	.00000219	-.00001067
6.000	4.91967984	.99999974	.00000134	-.00000666
6.100	5.01967982	.99999985	.00000081	-.00000411
6.200	5.11967981	.99999991	.00000049	-.00000251
6.300	5.21967981	.99999995	.00000029	-.00000152
6.400	5.31967980	.99999997	.00000017	-.00000090
6.500	5.41967980	.99999998	.00000010	-.00000053
6.580	5.49967980	.99999999	.00000006	-.00000034

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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    =  .2000000000

D2F(0)  =  .6867081800

 ETA          F             DF            D2F           D3F

 .000     .00000000     .00000000     .68670818    -.20000000
 .100     .00340018     .06766967     .66666262    -.20135094
 .200     .01346678     .13332379     .64635641    -.20514929
 .300     .02999653     .19692470     .62556404    -.21100888
 .400     .05278135     .25341414     .60409924    -.21852960
 .500     .08160649     .31771725     .58181673    -.22729117
 .600     .11624903     .37474677     .55861436    -.23685135
 .700     .15647689     .42940751     .53443521    -.24674860
 .800     .20204828     .48160087     .50926920    -.25650909
 .900     .25271157     .53122965     .48315381    -.26565757
1.000     .30820567     .57820275     .45617353    -.27373159
1.100     .36826090     .62243979     .42845780    -.28029799
1.200     .43260023     .66387544     .40017713    -.28497060
1.300     .50094103     .70246323     .37153762    -.28742752
1.400     .57299711     .73817879     .34277365    -.28742673
1.500     .64848103     .77102227     .31413931    -.28481831
1.600     .72710667     .80101978     .28589869    -.27955231
1.700     .80859184     .82822394     .25831580    -.27168107
1.800     .89266094     .85271334     .23164438    -.26135588
1.900     .97904743     .87459103     .20611847    -.24881786
2.000    1.06749623     .89398212     .18194407    -.23438380
2.100    1.15776575     .91103065     .15929247    -.21842800
2.200    1.24962957     .92589582     .13829551    -.20136154
2.300    1.34287781     .93874795     .11904297    -.18361062
2.400    1.43731796     .94976419     .10158216    -.16559547
2.500    1.53277544     .95912436     .08591947    -.14771134
2.600    1.62909359     .96700701     .07202365    -.13031276
2.700    1.72613339     .97358588     .05983060    -.11370170
2.800    1.82377285     .97902687     .04924893    -.09812013
2.900    1.92190604     .98348564     .04016622    -.08374690
3.000    2.02044204     .98710585     .03245532    -.07069850
3.100    2.11930362     .99001791     .02598036    -.05903318
3.200    2.21842592     .99233848     .02060231    -.04875757
3.300    2.31775504     .99417035     .01618371    -.03983494
3.400    2.41724669     .99560280     .01259256    -.03219434
3.500    2.51686485     .99671231     .00970529    -.02573981
3.600    2.61658055     .99756353     .00740880    -.02035911
3.700    2.71637075     .99821037     .00560169    -.01593149
3.800    2.81621730     .99869720     .00419482    -.01233430
3.900    2.91610607     .99906009     .00311114    -.00944820
4.000    3.01602616     .99932801     .00228523    -.00716101
4.100    3.11596928     .99952390     .00166240    -.00537038
4.200    3.21592915     .99966575     .00119765    -.00398523
4.300    3.31590109     .99976747     .00085449    -.00292640
4.400    3.41588166     .99983972     .00060375    -.00212645
4.500    3.51586832     .99989053     .00042246    -.00152909
4.600    3.61585925     .99992592     .00029273    -.00108811
4.700    3.71585314     .99995034     .00020087    -.00076628
4.800    3.81584906     .99996701     .00013650    -.00053405
4.900    3.91584637     .99997829     .00009185    -.00036836
5.000    4.01584460     .99998585     .00006120    -.00025145
```

(continued on next page)

BETA = .20 (continued)

5.100	4.11584345	.99999086	.00004039	-.00016988
5.200	4.21584271	.99999415	.00002639	-.00011359
5.300	4.31584224	.99999629	.00001707	-.00007518
5.400	4.41584195	.99999767	.00001094	-.00004924
5.500	4.51584176	.99999855	.00000694	-.00003192
5.600	4.61584165	.99999911	.00000436	-.00002049
5.700	4.71584158	.99999945	.00000271	-.00001301
5.800	4.81584153	.99999967	.00000167	-.00000818
5.900	4.91584151	.99999980	.00000102	-.00000509
6.000	5.01584149	.99999988	.00000062	-.00000314
6.100	5.11584148	.99999993	.00000037	-.00000192
6.200	5.21584148	.99999996	.00000022	-.00000116
6.300	5.31584147	.99999997	.00000013	-.00000070
6.400	5.41584147	.99999998	.00000008	-.00000042
6.500	5.51584147	.99999999	.00000005	-.00000026
6.510	5.52584147	.99999999	.00000004	-.00000021

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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    =   .3000000000

D2F(0) =   .7747545800

 ETA           F            DF           D2F          D3F

 .000      .00000000    .00000000    .77475458   -.30000000
 .100      .00382375    .07597450    .74471668   -.30111598
 .200      .01509450    .14893626    .71446813   -.30412993
 .300      .03350961    .21885555    .68384396   -.30854602
 .400      .05876274    .28568862    .65272864   -.31387073
 .500      .09054270    .34938264    .62105594   -.31961161
 .600      .12853274    .40988061    .58880873   -.32528056
 .700      .17241040    .46712622    .55601813   -.33040124
 .800      .22184786    .52106867    .52276192   -.33451985
 .900      .27651265    .57166709    .48916155   -.33721838
1.000      .33606891    .61889483    .45537791   -.33812911
1.100      .40017894    .66274302    .42160551   -.33694915
1.200      .46850523    .70322365    .38806536   -.33345360
1.300      .54071256    .74037180    .35499663   -.32750602
1.400      .61647046    .77424696    .32264738   -.31906507
1.500      .69545564    .80493351    .29126497   -.30818648
1.600      .77735448    .83254011    .26108645   -.29501981
1.700      .86186536    .85719822    .23232953   -.27980013
1.800      .94870088    .87905979    .20518467   -.26283504
1.900     1.03758973    .89829416    .17980863   -.24448787
2.000     1.12827823    .91508447    .15631985   -.22515841
2.100     1.22053158    .92962366    .13479582   -.20526251
2.200     1.31413455    .94211036    .11527248   -.18521197
2.300     1.40889191    .95274475    .09774556   -.16539616
2.400     1.50462836    .96172468    .08217365   -.14616650
2.500     1.60118811    .96924221    .06848261   -.12782441
2.600     1.69843417    .97548054    .05657100   -.11061343
2.700     1.79624732    .98061165    .04631609   -.09471540
2.800     1.89452490    .98479440    .03758009   -.08025041
2.900     1.99317943    .98817340    .03021613   -.06728016
3.000     2.09213713    .99087835    .02407387   -.05581387
3.100     2.19133647    .99302393    .01900436   -.04581600
3.200     2.29072662    .99471018    .01486408   -.03721503
3.300     2.39026608    .99602321    .01151806   -.02991255
3.400     2.48992127    .99703612    .00884216   -.02379198
3.500     2.58966535    .99781023    .00672448   -.01872659
3.600     2.68947706    .99839631    .00506601   -.01458635
3.700     2.78933974    .99883586    .00378064   -.01124357
3.800     2.88924047    .99916241    .00279476   -.00857708
3.900     2.98916935    .99940271    .00204641   -.00647533
4.000     3.08911885    .99957788    .00148422   -.00483816
4.100     3.18908331    .99970436    .00106624   -.00357768
4.200     3.28905852    .99979480    .00075866   -.00261839
4.300     3.38904139    .99985887    .00053465   -.00189664
4.400     3.48902966    .99990381    .00037318   -.00135975
4.500     3.58902170    .99993504    .00025798   -.00096487
4.600     3.68901635    .99995653    .00017663   -.00067766
4.700     3.78901278    .99997118    .00011977   -.00047109
4.800     3.88901042    .99998107    .00008043   -.00032414
4.900     3.98900888    .99998768    .00005349   -.00022076
5.000     4.08900789    .99999205    .00003523   -.00014882
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(continued on next page)

BETA = .30 (continued)

5.100	4.18900724	.99999492	.00002298	-.00009931
5.200	4.28900684	.99999679	.00001484	-.00006559
5.300	4.38900658	.99999799	.00000950	-.00004289
5.400	4.48900642	.99999875	.00000602	-.00002775
5.500	4.58900632	.99999923	.00000377	-.00001778
5.600	4.68900626	.99999953	.00000234	-.00001127
5.700	4.78900622	.99999972	.00000144	-.00000708
5.800	4.88900620	.99999983	.00000088	-.00000440
5.900	4.98900619	.99999990	.00000053	-.00000270
6.000	5.08900618	.99999994	.00000032	-.00000164
6.100	5.18900617	.99999996	.00000019	-.00000099
6.200	5.28900617	.99999998	.00000011	-.00000059
6.300	5.38900617	.99999999	.00000006	-.00000035
6.360	5.44900617	.99999999	.00000004	-.00000025

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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    =   .4000000000

D2F(0)  =   .8544212300

 ETA           F              DF             D2F             D3F

 .000      .00000000      .00000000      .85442123      -.40000000
 .100      .00420543      .08344156      .81439915      -.40063990
 .200      .01655475      .16287595      .77426239      -.40220629
 .300      .03664654      .23828793      .73394420      -.40418406
 .400      .06407761      .30965815      .69342840      -.40607797
 .500      .09844282      .37596805      .65274716      -.40741631
 .600      .13933543      .44020462      .61197840      -.40775823
 .700      .18634785      .49936477      .57124212      -.40670367
 .800      .23907285      .55445934      .53069587      -.40390491
 .900      .29710512      .60551656      .49052882      -.39907850
1.000      .36004317      .65258485      .45095479      -.39201640
1.100      .42749145      .69573496      .41220428      -.38259496
1.200      .49906267      .73506111      .37451566      -.37078085
1.300      .57438012      .77068140      .33812601      -.35663293
1.400      .65308011      .80273718      .30326201      -.34029959
1.500      .73481416      .83139161      .27013132      -.32201152
1.600      .81925112      .85682729      .23891488      -.30207008
1.700      .90607895      .87924336      .20976057      -.28083208
1.800      .99500620      .89885186      .18277856      -.25869193
1.900     1.08576310      .91587385      .15803849      -.23606240
2.000     1.17810228      .93053528      .13556864      -.21335536
2.100     1.27179903      .94306289      .11535687      -.19096371
2.200     1.36665119      .95368030      .09735339      -.16924568
2.300     1.46247866      .96260443      .08147485      -.14851231
2.400     1.55912255      .97004240      .06760954      -.12901865
2.500     1.65644411      .97518899      .05562326      -.11095884
2.600     1.75432334      .98122468      .04536550      -.09446501
2.700     1.85265753      .98531436      .03667558      -.07960954
2.800     1.95135965      .98860656      .02938834      -.06641006
2.900     2.05035668      .99123330      .02333933      -.05483657
3.000     2.14958800      .99331037      .01836911      -.04481981
3.100     2.24900379      .99493803      .01432677      -.03626028
3.200     2.34856350      .99620198      .01107249      -.02903709
3.300     2.44823449      .99717455      .00847926      -.02301638
3.400     2.54799072      .99791607      .00643378      -.01805862
3.500     2.64781167      .99847624      .00483674      -.01402486
3.600     2.74768128      .99889550      .00360247      -.01078156
3.700     2.84758716      .99920639      .00265824      -.00820422
3.800     2.94751981      .99943478      .00194322      -.00617972
3.900     3.04747205      .99960099      .00140723      -.00460765
4.000     3.14743847      .99972082      .00100953      -.00340074
4.100     3.24741508      .99980641      .00071741      -.00248458
4.200     3.34739892      .99986696      .00050501      -.00179689
4.300     3.44738787      .99990939      .00035213      -.00128643
4.400     3.54738037      .99993885      .00024321      -.00091168
4.500     3.64737533      .99995910      .00016638      -.00063959
4.600     3.74737197      .99997289      .00011274      -.00044418
4.700     3.84736976      .99998220      .00007567      -.00030536
4.800     3.94736831      .99998841      .00005030      -.00020782
4.900     4.04736737      .99999253      .00003312      -.00014001
5.000     4.14736677      .99999523      .00002159      -.00009338
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BETA = .40 (continued)

5.100	4.24736639	.99999698	.00001395	-.00006165
5.200	4.34736614	.99999810	.00000892	-.00004030
5.300	4.44736599	.99999882	.00000565	-.00002608
5.400	4.54736590	.99999927	.00000355	-.00001671
5.500	4.64736584	.99999955	.00000220	-.00001060
5.600	4.74736581	.99999973	.00000136	-.00000666
5.700	4.84736578	.99999983	.00000083	-.00000414
5.800	4.94736577	.99999990	.00000050	-.00000255
5.900	5.04736576	.99999994	.00000030	-.00000156
6.000	5.14736576	.99999996	.00000018	-.00000095
6.100	5.24736575	.99999997	.00000011	-.00000058
6.200	5.34736575	.99999998	.00000006	-.00000036
6.300	5.44736575	.99999998	.00000004	-.00000023
6.370	5.51736575	.99999999	.00000002	-.00000009

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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    =   .5000000000

D2F(0) =   .9276800400

 ETA          F              DF             D2F            D3F

 .000      .00000000      .00000000      .92768004      -.50000000
 .100      .00455507      .09026804      .87768195      -.49992374
 .200      .01788697      .17553722      .82771029      -.49939857
 .300      .03949606      .25581315      .77783135      -.49800109
 .400      .06888362      .33111011      .72815221      -.49534081
 .500      .10555300      .40145501      .67881699      -.49106810
 .600      .14901097      .46689083      .63000237      -.48488374
 .700      .19876957      .52747961      .58191200      -.47654903
 .800      .25434808      .58330483      .53476995      -.46589545
 .900      .31527528      .63447311      .48881331      -.45283269
1.000      .38109180      .68111507      .44428404      -.43735413
1.100      .45135255      .72338545      .40142047      -.41953889
1.200      .52562908      .76146225      .36044880      -.39954999
1.300      .60351185      .79554517      .32157497      -.37762825
1.400      .68461226      .82585316      .28497735      -.35408227
1.500      .76856447      .85262138      .25080061      -.32927483
1.600      .85502679      .87509757      .21915110      -.30360659
1.700      .94368279      .89553806      .19009397      -.27749816
1.800     1.03424191      .91420358      .16365205      -.25137172
1.900     1.12643971      .92935506      .13980665      -.22563335
2.000     1.22003770      .94224957      .11849992      -.20065724
2.100     1.31482272      .95313660      .09963876      -.17677261
2.200     1.41060607      .96225476      .08309980      -.15425397
2.300     1.50722223      .96982906      .06873524      -.13331507
2.400     1.60452741      .97506876      .05637903      -.11410659
2.500     1.70239791      .98116589      .04585322      -.09671717
2.600     1.80072832      .98529429      .03697388      -.08117750
2.700     1.89942968      .98860940      .02955671      -.06746662
2.800     1.99842768      .99124836      .02342171      -.05551994
2.900     2.09766082      .99333074      .01839720      -.04523810
3.000     2.19707872      .99495946      .01432275      -.03649605
3.100     2.29664052      .99622205      .01105138      -.02915187
3.200     2.39631340      .99719208      .00845080      -.02305476
3.300     2.49607124      .99793065      .00640396      -.01805195
3.400     2.59589349      .99848791      .00480892      -.01399438
3.500     2.69576414      .99890456      .00357829      -.01074105
3.600     2.79567081      .99921324      .00263824      -.00816210
3.700     2.89560406      .99943983      .00192730      -.00614070
3.800     2.99555673      .99960464      .00139496      -.00457396
3.900     3.09552346      .99972341      .00100032      -.00337308
4.000     3.19550028      .99980820      .00071067      -.00246274
4.100     3.29548427      .99986818      .00050019      -.00178020
4.200     3.39547332      .99991020      .00034877      -.00127402
4.300     3.49546588      .99993938      .00024090      -.00090270
4.400     3.59546089      .99995944      .00016484      -.00063323
4.500     3.69545756      .99997311      .00011173      -.00043979
4.600     3.79545536      .99998233      .00007502      -.00030240
4.700     3.89545393      .99998850      .00004989      -.00020586
4.800     3.99545299      .99999258      .00003287      -.00013875
4.900     4.09545239      .99999526      .00002145      -.00009258
5.000     4.19545201      .99999700      .00001386      -.00006116
5.100     4.29545177      .99999812      .00000887      -.00004000
5.200     4.39545162      .99999883      .00000563      -.00002590
5.300     4.49545153      .99999928      .00000353      -.00001660
5.400     4.59545147      .99999956      .00000220      -.00001054
5.500     4.69545144      .99999973      .00000135      -.00000662
5.600     4.79545142      .99999984      .00000082      -.00000411
5.700     4.89545140      .99999991      .00000050      -.00000253
5.800     4.99545140      .99999995      .00000030      -.00000153
5.900     5.09545139      .99999997      .00000017      -.00000091
6.000     5.19545139      .99999998      .00000010      -.00000054
6.080     5.27545139      .99999999      .00000006      -.00000034
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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    =   .6000000000

D2F(0) =   .9958364400

 ETA            F              DF             D2F             D3F

 .000       .00000000      .00000000      .99583644      -.60000000
 .100       .00487920      .09558449      .93587048      -.59896916
 .200       .01911728      .18718113      .87611635      -.59572689
 .300       .04211689      .27182252      .81680567      -.59006882
 .400       .07328514      .35056538      .75818878      -.58182632
 .500       .11203607      .42349223      .70053068      -.57087730
 .600       .15779334      .49071262      .64410580      -.55715628
 .700       .20999292      .55236376      .58919192      -.54066270
 .800       .26808591      .60861052      .53606341      -.52146699
 .900       .33154124      .65964476      .48498387      -.49971343
1.000       .39984833      .70568381      .43619881      -.47561958
1.100       .47251950      .74696838      .38992849      -.44947176
1.200       .54909221      .78375966      .34636142      -.42161684
1.300       .62913092      .81633586      .30564887      -.39245061
1.400       .71222858      .84498829      .26790068      -.36240339
1.500       .79800778      .87001705      .23318261      -.33192373
1.600       .88612135      .89172656      .20151534      -.30146130
1.700       .97625260      .91042107      .17287527      -.27145002
1.800      1.06811507      .92640037      .14719683      -.24229256
1.900      1.16145189      .93995574      .12437630      -.21434702
2.000      1.25603474      .95136637      .10427685      -.18791657
2.100      1.35166249      .96089637      .08673437      -.16324250
2.200      1.44815957      .96879232      .07156387      -.14050076
2.300      1.54537409      .97528157      .05856605      -.11980197
2.400      1.64317591      .98057105      .04753370      -.10119448
2.500      1.74145453      .98484685      .03825767      -.08466999
2.600      1.84011702      .98827426      .03053214      -.07017110
2.700      1.93908596      .99099835      .02415915      -.05760003
2.800      2.03829746      .99314494      .01895214      -.04682798
2.900      2.13769931      .99482188      .01473859      -.03770434
3.000      2.23724925      .99612053      .01136178      -.03006546
3.100      2.33691338      .99711744      .00868164      -.02374234
3.200      2.43666480      .99787596      .00657504      -.01856732
3.300      2.53648236      .99844799      .00493529      -.01437925
3.400      2.63634959      .99887553      .00367133      -.01102752
3.500      2.73625378      .99919221      .00270651      -.00837466
3.600      2.83618523      .99942467      .00197721      -.00629794
3.700      2.93613661      .99959376      .00143132      -.00468993
3.800      3.03610241      .99971563      .00102669      -.00345834
3.900      3.13607858      .99980268      .00072971      -.00252521
4.000      3.23606210      .99986427      .00051387      -.00182579
4.100      3.33605082      .99990746      .00035854      -.00130715
4.200      3.43604316      .99993746      .00024785      -.00092666
4.300      3.53603800      .99995811      .00016974      -.00065047
4.400      3.63603456      .99997219      .00011517      -.00045212
4.500      3.73603229      .99998171      .00007741      -.00031116
4.600      3.83603080      .99998807      .00005155      -.00021204
4.700      3.93602983      .99999229      .00003400      -.00014308
4.800      4.03602921      .99999506      .00002222      -.00009559
4.900      4.13602881      .99999687      .00001438      -.00006324
5.000      4.23602856      .99999803      .00000922      -.00004142
5.100      4.33602840      .99999877      .00000586      -.00002687
5.200      4.43602831      .99999924      .00000368      -.00001725
5.300      4.53602825      .99999953      .00000230      -.00001097
5.400      4.63602821      .99999971      .00000142      -.00000691
5.500      4.73602819      .99999983      .00000087      -.00000431
5.600      4.83602817      .99999989      .00000052      -.00000267
5.700      4.93602817      .99999993      .00000032      -.00000163
5.800      5.03602816      .99999996      .00000019      -.00000100
5.900      5.13602816      .99999997      .00000011      -.00000060
6.000      5.23602815      .99999998      .00000007      -.00000037
6.100      5.33602815      .99999999      .00000004      -.00000023
6.170      5.40602815      .99999999      .00000002      -.00000011
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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA   =  .7000000000

D2F(0) = 1.0598077730

 ETA          F              DF             D2F            D3F

 .000      .00000000      .00000000     1.05980777     -.70000000
 .100      .00518241      .10248264      .98988204     -.69777809
 .200      .02026401      .19799111      .92039702     -.69121059
 .300      .04455030      .28559098      .85177962     -.68045296
 .400      .07735545      .36838966      .78444020     -.66568307
 .500      .11800640      .44353468      .71876971     -.64711332
 .600      .16584674      .51221152      .65513573     -.62499968
 .700      .22024042      .57464106      .59387780     -.59964725
 .800      .28057512      .63107654      .53530242     -.57141222
 .900      .34626530      .68179996      .47967797     -.54070001
1.000      .41675491      .72711807      .42723006     -.50795975
1.100      .49151962      .76735787      .37813752     -.47367534
1.200      .57006861      .80286178      .33252957     -.43835374
1.300      .65194587      .83398259      .29048413     -.40251106
1.400      .73673096      .86107830      .25202770     -.36665752
1.500      .82403930      .88450703      .21713665     -.33128225
1.600      .91352192      .90462216      .18574013     -.29683881
1.700     1.00486476      .92176779      .15772425     -.26373245
1.800     1.09778754      .93527468      .13293744     -.23230987
1.900     1.19204223      .94845684      .11119671     -.20285192
2.000     1.28741125      .95360866      .09229446     -.17556954
2.100     1.38370540      .96700285      .07600553     -.15060310
2.200     1.48076158      .97388902      .06209417     -.12802478
2.300     1.57844049      .97949294      .05032061     -.10784359
2.400     1.67662418      .98401647      .04044706     -.09001262
2.500     1.77521374      .98763799      .03224284     -.07443777
2.600     1.87412693      .99051335      .02548866     -.06098729
2.700     1.97329605      .99277721      .01997987     -.04950158
2.800     2.07266585      .99454456      .01552877     -.03980273
2.900     2.17219167      .99591256      .01176601     -.03170319
3.000     2.27183777      .99596234      .00914114     -.02501346
3.100     2.37157578      .99776097      .00692251     -.01954838
3.200     2.47138343      .99836324      .00519652     -.01513217
3.300     2.57124337      .99881346      .00386655     -.01160202
3.400     2.67114224      .99914704      .00285151     -.00881044
3.500     2.77106983      .99939201      .00208423     -.00662648
3.600     2.87101842      .99957030      .00150978     -.00493607
3.700     2.97098224      .99969890      .00108384     -.00364153
3.800     3.07095698      .99979083      .00077104     -.00266063
3.900     3.17093951      .99985595      .00054354     -.00192520
4.000     3.27092753      .99990165      .00037969     -.00137959
4.100     3.37091938      .99993345      .00026280     -.00097905
4.200     3.47091389      .99995535      .00018023     -.00068806
4.300     3.57091022      .99997032      .00012247     -.00047887
4.400     3.67090779      .99998044      .00008245     -.00033005
4.500     3.77090620      .99998722      .00005499     -.00022526
4.600     3.87090516      .99999173      .00003634     -.00015225
4.700     3.97090449      .99999469      .00002379     -.00010190
4.800     4.07090406      .99999663      .00001543     -.00006753
4.900     4.17090379      .99999787      .00000991     -.00004432
5.000     4.27090362      .99999867      .00000631     -.00002880
5.100     4.37090352      .99999918      .00000398     -.00001854
5.200     4.47090345      .99999949      .00000248     -.00001181
5.300     4.57090341      .99999969      .00000154     -.00000745
5.400     4.67090339      .99999981      .00000094     -.00000465
5.500     4.77090337      .99999989      .00000057     -.00000288
5.600     4.87090337      .99999993      .00000034     -.00000176
5.700     4.97090336      .99999996      .00000020     -.00000106
5.800     5.07090336      .99999998      .00000012     -.00000063
5.900     5.17090335      .99999998      .00000007     -.00000037
6.000     5.27090335      .99999999      .00000004     -.00000021
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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    =   .8000000000

D2F(0)  =  1.1202676570

  ETA           F              DF            D2F            D3F

  .000      .00000000      .00000000     1.12026766     -.80000000
  .100      .00546807      .10802984     1.04039021     -.79635257
  .200      .02134063      .20810186      .96122501     -.78586804
  .300      .04682659      .30032033      .88342224     -.76921380
  .400      .08114839      .38485127      .80756610     -.74704429
  .500      .12354791      .46191581      .73417546     -.72001287
  .600      .17329161      .53178373      .66370361     -.68877912
  .700      .22967512      .59476680      .59653743     -.65401177
  .800      .29202702      .65121213      .53299649     -.61638759
  .900      .35971212      .70149540      .47333246     -.57658678
 1.000      .43213393      .74501405      .41772914     -.53528536
 1.100      .50873651      .78518055      .36630332     -.49314508
 1.200      .58900566      .81941576      .31910684     -.45080198
 1.300      .67246939      .84914263      .27612969     -.40885421
 1.400      .75869788      .87478016      .23730447     -.36785014
 1.500      .84730278      .89573799      .20251187     -.32827765
 1.600      .93793602      .91541150      .17158719     -.29055523
 1.700     1.03028819      .93117763      .14432764     -.25502564
 1.800     1.12408650      .94439146      .12050010     -.22195235
 1.900     1.21909246      .95538355      .09984908     -.19151908
 2.000     1.31509933      .96445817      .08210466     -.16383213
 2.100     1.41192943      .97189214      .06698991     -.13892555
 2.200     1.50943140      .97793456      .05422784     -.11676839
 2.300     1.60747738      .98280708      .04354743     -.09727369
 2.400     1.70596035      .98570476      .03468874     -.08030859
 2.500     1.80479152      .98979738      .02740697     -.06570479
 2.600     1.90389789      .99223114      .02147549     -.05326904
 2.700     2.00321996      .99413058      .01668784     -.04279292
 2.800     2.10270971      .99560063      .01285875     -.03406172
 2.900     2.20232871      .99572880      .00982445     -.02686203
 3.000     2.30204649      .99758724      .00744217     -.02098798
 3.100     2.40183914      .99823485      .00558914     -.01624595
 3.200     2.50168803      .99871921      .00416120     -.01245796
 3.300     2.60157882      .99907834      .00307111     -.00946371
 3.400     2.70150053      .99934228      .00224674     -.00712157
 3.500     2.80144489      .99953456      .00162918     -.00530857
 3.600     2.90140567      .99967341      .00117090     -.00391972
 3.700     3.00137826      .99977278      .00083405     -.00286681
 3.800     3.10135927      .99984327      .00058879     -.00207681
 3.900     3.20134622      .99989282      .00041192     -.00149019
 4.000     3.30133733      .99992733      .00028558     -.00105907
 4.100     3.40133133      .99995116      .00019620     -.00074548
 4.200     3.50132731      .99996746      .00013357     -.00051972
 4.300     3.60132465      .99997851      .00009010     -.00035886
 4.400     3.70132289      .99998593      .00006022     -.00024540
 4.500     3.80132175      .99999087      .00003988     -.00016620
 4.600     3.90132101      .99999413      .00002617     -.00011148
 4.700     4.00132054      .99999626      .00001701     -.00007406
 4.800     4.10132024      .99999763      .00001096     -.00004872
 4.900     4.20132005      .99999852      .00000699     -.00003174
 5.000     4.30131993      .99999908      .00000442     -.00002048
 5.100     4.40131986      .99999943      .00000277     -.00001309
 5.200     4.50131981      .99999965      .00000172     -.00000829
 5.300     4.60131979      .99999979      .00000106     -.00000520
 5.400     4.70131977      .99999987      .00000064     -.00000323
 5.500     4.80131976      .99999992      .00000039     -.00000199
 5.600     4.90131975      .99999995      .00000023     -.00000122
 5.700     5.00131975      .99999997      .00000014     -.00000075
 5.800     5.10131975      .99999998      .00000008     -.00000046
 5.900     5.20131975      .99999998      .00000005     -.00000029
 6.000     5.30131974      .99999999      .00000003     -.00000019
 6.050     5.35131974      .99999999      .00000001     -.00000007
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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    =  .9000000000

D2F(0)  = 1.1777278190

 ETA           F              DF             D2F            D3F

 .000       .00000000      .00000000     1.17772782     -.90000000
 .100       .00573873      .11327728     1.08790667     -.89469463
 .200       .02235739      .21761568      .99911121     -.87971659
 .300       .04896876      .31316384      .91224099     -.85640688
 .400       .08470480      .40015383      .82806533     -.82603033
 .500       .12872429      .47388840      .74723045     -.78978602
 .600       .18021930      .54972905      .67026568     -.74881299
 .700       .23842055      .61308460      .59758938     -.70419213
 .800       .30260153      .66940042      .52951474     -.65694474
 .900       .37208168      .71914818      .46625598     -.60802861
1.000       .44622851      .76281630      .40793493     -.55833236
1.100       .52445883      .80090107      .35458833     -.50866871
1.200       .60623915      .83389851      .30617567     -.45976763
1.300       .69108525      .86229710      .26258791     -.41226998
1.400       .77856111      .88557136      .22365657     -.36672241
1.500       .86827724      .90717639      .18916335     -.32357412
1.600       .95988849      .92454339      .15884997     -.28317582
1.700      1.05309148      .93907612      .13242796     -.24578119
1.800      1.14762173      .95114840      .10958816     -.21155080
1.900      1.24325059      .96110247      .09000977     -.18055854
2.000      1.33978199      .96924826      .07336859     -.15279994
2.100      1.43704925      .97586341      .05934435     -.12820231
2.200      1.53491188      .98119401      .04762704     -.10663579
2.300      1.63325246      .98545587      .03792196     -.08792519
2.400      1.73197371      .98883627      .02995374     -.07186164
2.500      1.83099572      .99149604      .02346909     -.05821385
2.600      1.93025347      .99357187      .01823855     -.04673848
2.700      2.02969448      .99517870      .01405720     -.03718924
2.800      2.12927679      .99641224      .01074457     -.02932454
2.900      2.22896713      .99735132      .00814385     -.02291368
3.000      2.32873939      .99806023      .00612056     -.01774140
3.100      2.42857325      .99859086      .00456086     -.01361104
3.200      2.52845301      .99898466      .00336952     -.01034636
3.300      2.62836672      .99927441      .00246793     -.00779220
3.400      2.72830529      .99948575      .00179190     -.00581425
3.500      2.82826192      .99963857      .00128970     -.00429807
3.600      2.92823156      .99974810      .00092011     -.00314765
3.700      3.02821049      .99982592      .00065064     -.00228360
3.800      3.12819598      .99988072      .00045602     -.00164120
3.900      3.22818608      .99991897      .00031676     -.00116842
4.000      3.32817938      .99994542      .00021806     -.00082399
4.100      3.42817489      .99996356      .00014877     -.00057560
4.200      3.52817190      .99997588      .00010058     -.00039828
4.300      3.62816993      .99998417      .00006738     -.00027297
4.400      3.72816865      .99998970      .00004473     -.00018531
4.500      3.82816781      .99999336      .00002943     -.00012460
4.600      3.92816728      .99999576      .00001918     -.00008298
4.700      4.02816694      .99999731      .00001239     -.00005473
4.800      4.12816672      .99999831      .00000792     -.00003575
4.900      4.22816659      .99999895      .00000502     -.00002313
5.000      4.32816650      .99999935      .00000315     -.00001482
5.100      4.42816645      .99999960      .00000196     -.00000940
5.200      4.52816642      .99999976      .00000121     -.00000590
5.300      4.62816640      .99999985      .00000073     -.00000366
5.400      4.72816639      .99999991      .00000044     -.00000225
5.500      4.82816638      .99999995      .00000026     -.00000136
5.600      4.92816638      .99999997      .00000015     -.00000080
5.700      5.02816637      .99999998      .00000008     -.00000046
5.800      5.12816637      .99999999      .00000004     -.00000025
5.860      5.18816637      .99999999      .00000003     -.00000016
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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    = 1.0000000000

D2F(0)  = 1.2325876570

 ETA           F             DF            D2F           D3F

  .000      .00000000     .00000000    1.23258766   -1.00000000
  .100      .00599639     .11326489    1.13283076    -.99280632
  .200      .02332226     .22561242    1.03445419    -.97277262
  .300      .05099480     .32524108     .93863123    -.94208355
  .400      .08805659     .41445611     .84632542    -.90275066
  .500      .13358522     .49464929     .75830748    -.85662075
  .600      .18670099     .56628052     .67517141    -.80538155
  .700      .24657292     .62986090     .59735023    -.75056563
  .800      .31242302     .68593745     .52513134    -.69355293
  .900      .38352924     .73507927     .45867173    -.63557248
 1.000      .45922702     .77786527     .39801295    -.57770392
 1.100      .53890971     .81487335     .34309599    -.52087918
 1.200      .62202803     .84567111     .29377595    -.46588490
 1.300      .70808860     .87380796     .24983635    -.41336593
 1.400      .79665178     .89580865     .21100317    -.36383030
 1.500      .88732899     .91516823     .17695812    -.31765584
 1.600      .97977948     .93234823     .14735130    -.27509856
 1.700     1.07370688     .94577411     .12181291    -.23630269
 1.800     1.16885548     .95583379     .09996382    -.20131235
 1.900     1.26500647     .96587721     .08142495    -.17008431
 2.000     1.36197416     .97321674     .06582538    -.14250163
 2.100     1.45960226     .97912837     .05280895    -.11838770
 2.200     1.55776032     .98385341     .04203959    -.09752008
 2.300     1.65634039     .98760076     .03320520    -.07964386
 2.400     1.75525388     .99054940     .02602026    -.06448405
 2.500     1.85442873     .99285118     .02022731    -.05175665
 2.600     1.95380682     .99463360     .01559731    -.04117815
 2.700     2.05334169     .99600267     .01192924    -.03247347
 2.800     2.15299651     .99704567     .00904886    -.02538210
 2.900     2.25274234     .99783371     .00680711    -.01966256
 3.000     2.35255668     .99842416     .00507796    -.01509540
 3.100     2.45242213     .99886286     .00375618    -.01148472
 3.200     2.55232541     .99918606     .00275489    -.00865861
 3.300     2.65225644     .99942214     .00200328    -.00646858
 3.400     2.75220767     .99959312     .00144421    -.00478835
 3.500     2.85217346     .99971588     .00103216    -.00351207
 3.600     2.95214967     .99980325     .00073126    -.00255226
 3.700     3.05213326     .99986489     .00051355    -.00183762
 3.800     3.15212203     .99990801     .00035749    -.00131082
 3.900     3.25211442     .99993790     .00024665    -.00092635
 4.000     3.35210930     .99995843     .00016867    -.00064854
 4.100     3.45210589     .99997242     .00011432    -.00044980
 4.200     3.55210363     .99998185     .00007678    -.00030904
 4.300     3.65210216     .99998817     .00005111    -.00021033
 4.400     3.75210120     .99999235     .00003371    -.00014180
 4.500     3.85210058     .99999510     .00002204    -.00009469
 4.600     3.95210018     .99999689     .00001427    -.00006264
 4.700     4.05209993     .99999804     .00000916    -.00004104
 4.800     4.15209978     .99999878     .00000583    -.00002663
 4.900     4.25209968     .99999925     .00000367    -.00001711
 5.000     4.35209962     .99999954     .00000229    -.00001089
 5.100     4.45209959     .99999972     .00000142    -.00000686
 5.200     4.55209956     .99999984     .00000087    -.00000427
 5.300     4.65209955     .99999990     .00000052    -.00000263
 5.400     4.75209954     .99999995     .00000031    -.00000160
 5.500     4.85209954     .99999997     .00000018    -.00000095
 5.600     4.95209954     .99999999     .00000011    -.00000055
 5.640     4.99209954     .99999999     .00000008    -.00000043
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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA    = 1.2000000000

D2F(0)  = 1.3357214750

 ETA          F              DF             D2F            D3F

 .000      .00000000      .00000000     1.33572147    -1.20000000
 .100      .00647881      .12758213     1.21611679    -1.18834635
 .200      .02512065      .24329755     1.09872248    -1.15656818
 .300      .05475306      .34746037      .98532644    -1.10907519
 .400      .09424322      .44054224      .87730009    -1.04978663
 .500      .14251172      .52313319      .77564730     -.98213682
 .600      .19854257      .59590726      .68105209     -.90909128
 .700      .26139019      .65959279      .59392441     -.83317084
 .800      .33018342      .71494723      .51444314     -.75648114
 .900      .40412745      .76273606      .44259599     -.68074564
1.000      .48250368      .80371547      .37821599     -.60734034
1.100      .56466805      .83361864      .32101448     -.53732915
1.200      .65004823      .86814502      .27061053     -.47149895
1.300      .73813980      .89295247      .22655695     -.41039376
1.400      .82850184      .91365176      .18836294     -.35434760
1.500      .92075195      .93080322      .15551346     -.30351576
1.600     1.01456122      .94491517      .12748564     -.25790438
1.700     1.10964893      .95544381      .10376231     -.21739802
1.800     1.20577744      .96579440      .08384284     -.18178522
1.900     1.30274714      .97332329      .06725156     -.15078191
2.000     1.40039176      .97934079      .05354396     -.12405245
2.100     1.49857388      .98411452      .04231087     -.10122831
2.200     1.59718086      .98787303      .03318096     -.08192424
2.300     1.69612113      .99080969      .02582164     -.06575202
2.400     1.79532084      .99308653      .01993884     -.05233159
2.500     1.89472095      .99483807      .01527572     -.04129988
2.600     1.99427465      .99517491      .01161059     -.03231738
2.700     2.09394513      .99718712      .00875436     -.02507256
2.800     2.19370369      .99794740      .00654755     -.01928459
2.900     2.29352816      .99851382      .00485722     -.01470435
3.000     2.39340154      .99893238      .00357371     -.01111426
3.100     2.49331092      .99923913      .00260764     -.00832705
3.200     2.59324658      .99946207      .00188687     -.00618381
3.300     2.69320126      .99962274      .00135387     -.00455149
3.400     2.79316960      .99973757      .00096323     -.00332019
3.500     2.89314767      .99981894      .00067947     -.00240030
3.600     2.99313259      .99987611      .00047521     -.00171967
3.700     3.09312231      .99991593      .00032949     -.00122091
3.800     3.19311537      .99994343      .00022648     -.00085894
3.900     3.29311071      .99996225      .00015432     -.00059878
4.000     3.39310762      .99997502      .00010423     -.00041360
4.100     3.49310558      .99998362      .00006978     -.00028307
4.200     3.59310424      .99998934      .00004631     -.00019195
4.300     3.69310338      .99999313      .00003046     -.00012897
4.400     3.79310283      .99999561      .00001985     -.00008584
4.500     3.89310247      .99999722      .00001283     -.00005661
4.600     3.99310225      .99999825      .00000821     -.00003699
4.700     4.09310211      .99999891      .00000521     -.00002394
4.800     4.19310203      .99999933      .00000328     -.00001535
4.900     4.29310197      .99999959      .00000204     -.00000975
5.000     4.39310194      .99999976      .00000126     -.00000614
5.100     4.49310192      .99999985      .00000078     -.00000383
5.200     4.59310191      .99999992      .00000047     -.00000237
5.300     4.69310190      .99999995      .00000029     -.00000146
5.400     4.79310190      .99999997      .00000017     -.00000089
5.500     4.89310190      .99999999      .00000011     -.00000055
5.510     4.90310190      .99999999      .00000009     -.00000047
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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA   = 1.6000000000

D2F(0) = 1.5215139959

 ETA          F              DF            D2F           D3F

 .000      .00000000      .00000000     1.52151400    -1.60000000
 .100      .00734131      .14417151     1.36230727    -1.57674444
 .200      .02830935      .27260734     1.20743691    -1.51527814
 .300      .06135813      .38591285     1.06015129    -1.42676293
 .400      .10501663      .48496538      .92267100    -1.32058952
 .500      .15791118      .57082028      .79636488    -1.20441764
 .600      .21877928      .64463423      .68191727    -1.08430410
 .700      .28647658      .70760451      .57947969     -.96488072
 .800      .35997849      .76092253      .48880344     -.84955376
 .900      .43837779      .80573958      .40935208     -.74070483
1.000      .52087937      .84314286      .34039426     -.63988052
1.100      .60679292      .87413990      .28107850     -.54796354
1.200      .69552454      .89964955      .23049194     -.46532189
1.300      .78656757      .92049826      .18770529     -.39193561
1.400      .87949339      .93742015      .15180618     -.32750207
1.500      .97394229      .95105997      .12192296     -.27152183
1.600     1.06961475      .96197804      .09724087     -.22336747
1.700     1.16626333      .97065650      .07701195     -.18233775
1.800     1.26368516      .97750626      .06056027     -.14769955
1.900     1.36171525      .98287430      .04728346     -.11871958
2.000     1.46022037      .98705103      .03665147     -.09468764
2.100     1.55909382      .99027732      .02820341     -.07493308
2.200     1.65825079      .99275123      .02154301     -.05883569
2.300     1.75762440      .99463422      .01633321     -.04583209
2.400     1.85716232      .99605672      .01229031     -.03541873
2.500     1.95682391      .99712325      .00917797     -.02715203
2.600     2.05657789      .99791680      .00680126     -.02064663
2.700     2.15640036      .99850268      .00500101     -.01557200
2.800     2.25627321      .99893190      .00364855     -.01164823
2.900     2.35618284      .99924386      .00264086     -.00864107
3.000     2.45611909      .99946882      .00189628     -.00635681
3.100     2.55607448      .99962974      .00135071     -.00463714
3.200     2.65604349      .99974392      .00095433     -.00335408
3.300     2.75602214      .99982429      .00066877     -.00240539
3.400     2.85600754      .99988039      .00046482     -.00171026
3.500     2.95599764      .99991923      .00032039     -.00120554
3.600     3.05599098      .99994589      .00021900     -.00084240
3.700     3.15598654      .99996405      .00014845     -.00058352
3.800     3.25598360      .99997631      .00009977     -.00040066
3.900     3.35598167      .99998452      .00006649     -.00027269
4.000     3.45598041      .99998996      .00004393     -.00018395
4.100     3.55597960      .99999355      .00002878     -.00012299
4.200     3.65597908      .99999589      .00001869     -.00008150
4.300     3.75597875      .99999740      .00001203     -.00005352
4.400     3.85597854      .99999837      .00000768     -.00003484
4.500     3.95597841      .99999899      .00000486     -.00002247
4.600     4.05597833      .99999937      .00000305     -.00001437
4.700     4.15597828      .99999962      .00000190     -.00000911
4.800     4.25597825      .99999977      .00000117     -.00000573
4.900     4.35597823      .99999986      .00000072     -.00000358
5.000     4.45597822      .99999991      .00000044     -.00000222
5.100     4.55597821      .99999995      .00000027     -.00000138
5.200     4.65597821      .99999997      .00000016     -.00000086
5.300     4.75597821      .99999998      .00000010     -.00000055
5.370     4.82597821      .99999999      .00000005     -.00000026
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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA   = 2.0000000000

D2F(0) = 1.6872181692

 ETA          F            DF           D2F          D3F

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 .100     .00810343    .15375520   1.48853187  -1.96165579
 .200     .03109816    .29794410   1.29689992  -1.86278982
 .300     .06707197    .41353771   1.11728031  -1.72459056
 .400     .11423121    .52190386    .95272163  -1.56406327
 .500     .17093154    .60963696    .80477340  -1.39424670
 .600     .23569384    .68342727    .67386074  -1.22467915
 .700     .30720865    .74496498    .55960634  -1.06197027
 .800     .38433266    .79587364    .46109394   -.91038377
 .900     .46607969    .83766709    .37707552   -.77237494
1.000     .55160838    .87172451    .30612824   -.64905567
1.100     .64020800    .89927898    .24676894   -.54057809
1.200     .73128374    .92141570    .19753419   -.44643977
1.300     .82434210    .93907683    .15703348   -.36571872
1.400     .91897699    .95307056    .12398193   -.29724955
1.500    1.01485694    .96408265    .09721785   -.23975148
1.600    1.11171345    .97268918    .07570950   -.19191879
1.700    1.20933067    .97936952    .05855430   -.15248220
1.800    1.30753640    .98451904    .04497307   -.12024844
1.900    1.40619429    .98846099    .03430110   -.09412374
2.000    1.50519714    .99145744    .02597757   -.07312564
2.100    1.60446133    .99371909    .01953414   -.05638651
2.200    1.70392211    .99541395    .01458358   -.04315141
2.300    1.80352969    .99667493    .01080877   -.03277211
2.400    1.90324613    .99760627    .00795239   -.02469883
2.500    2.00304268    .99828908    .00580759   -.01847067
2.600    2.10289776    .99878597    .00420957   -.01370546
2.700    2.20279529    .99914485    .00302825   -.01008973
2.800    2.30272336    .99940209    .00216184   -.00736904
2.900    2.40267324    .99958507    .00153146   -.00533898
3.000    2.50263858    .99971422    .00107648   -.00383702
3.100    2.60261480    .99980466    .00075075   -.00273520
3.200    2.70259860    .99986750    .00051945   -.00193382
3.300    2.80258765    .99991082    .00035656   -.00135597
3.400    2.90258031    .99994045    .00024278   -.00094290
3.500    3.00257542    .99996054    .00016398   -.00065018
3.600    3.10257220    .99997406    .00010985   -.00044457
3.700    3.20257009    .99998309    .00007299   -.00030141
3.800    3.30256871    .99998906    .00004810   -.00020261
3.900    3.40256783    .99999298    .00003143   -.00013503
4.000    3.50256726    .99999553    .00002037   -.00008921
4.100    3.60256691    .99999718    .00001309   -.00005843
4.200    3.70256668    .99999823    .00000834   -.00003794
4.300    3.80256654    .99999890    .00000526   -.00002441
4.400    3.90256645    .99999932    .00000329   -.00001557
4.500    4.00256640    .99999958    .00000204   -.00000984
4.600    4.10256636    .99999975    .00000125   -.00000616
4.700    4.20256634    .99999985    .00000076   -.00000381
4.800    4.30256633    .99999991    .00000045   -.00000233
4.900    4.40256632    .99999994    .00000027   -.00000140
5.000    4.50256632    .99999996    .00000015   -.00000082
5.100    4.60256632    .99999998    .00000008   -.00000046
5.200    4.70256631    .99999999    .00000004   -.00000023
5.300    4.80256631    .99999999    .00000001   -.00000009
5.320    4.82256631    .99999999    .00000001   -.00000006
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FALKNER AND SKAN EQUATION NUMERICAL SOLUTION

BETA   = 2.3999999999

D2F(0) = 1.8381744506

 ETA          F            DF           D2F          D3F

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 .100      .00879188    .17186721   1.60012861   -2.34317613
 .200      .03359382    .32037448   1.37241659   -2.19976917
 .300      .07213438    .44692700   1.16189688   -2.00442772
 .400      .12231149    .55345603    .97236639   -1.78377901
 .500      .18223108    .64215203    .80533536   -1.55709498
 .600      .25022274    .71527101    .66070721   -1.33745368
 .700      .32483921    .77500262    .53733308   -1.13303711
 .800      .40484526    .82338730    .43343906    -.94835580
 .900      .48920019    .86227033    .34693917    -.78529842
1.000      .57703719    .89328222    .27565368    -.64397493
1.100      .66764163    .91783707    .21745315    -.52336104
1.200      .76042984    .93714245    .17034700    -.42177062
1.300      .85492925    .95221594    .13253196    -.33718897
1.400      .95076038    .96390514    .10241288    -.26749864
1.500     1.04762089    .97290867    .07860515    -.21062532
1.600     1.14527172    .97979690    .05992555    -.16462632
1.700     1.24352525    .98503128    .04537640    -.12773881
1.800     1.34223528    .98898199    .03412636    -.09840072
1.900     1.44128874    .99194354    .02549010    -.07525384
2.000     1.54059881    .99414837    .01890818    -.05713558
2.100     1.64009930    .99577847    .01392826    -.04306429
2.200     1.73974010    .99697525    .01018788    -.03222108
2.300     1.83948357    .99784770    .00739910    -.02393044
2.400     1.93930163    .99847918    .00533520    -.01764094
2.500     2.03917350    .99893296    .00381916    -.01290699
2.600     2.13908390    .99925667    .00271392    -.00937196
2.700     2.23902169    .99948590    .00191429    -.00675317
2.800     2.33897882    .99964702    .00134020    -.00482867
2.900     2.43894948    .99975943    .00093121    -.00342579
3.000     2.53892955    .99983725    .00064212    -.00241144
3.100     2.63891612    .99989072    .00043939    -.00168402
3.200     2.73890713    .99992718    .00029834    -.00116666
3.300     2.83890117    .99995184    .00020099    -.00080175
3.400     2.93889723    .99996839    .00013434    -.00054652
3.500     3.03889466    .99997942    .00008909    -.00036951
3.600     3.13889299    .99998670    .00005860    -.00024778
3.700     3.23889192    .99999148    .00003824    -.00016478
3.800     3.33889123    .99999458    .00002475    -.00010867
3.900     3.43889080    .99999658    .00001589    -.00007107
4.000     3.53889052    .99999786    .00001012    -.00004608
4.100     3.63889035    .99999867    .00000639    -.00002962
4.200     3.73889025    .99999918    .00000400    -.00001888
4.300     3.83889018    .99999950    .00000249    -.00001192
4.400     3.93889015    .99999970    .00000153    -.00000746
4.500     4.03889012    .99999982    .00000093    -.00000462
4.600     4.13889011    .99999990    .00000056    -.00000283
4.700     4.23889010    .99999994    .00000034    -.00000170
4.800     4.33889010    .99999997    .00000020    -.00000100
4.900     4.43889009    .99999999    .00000011    -.00000056
4.940     4.47889009    .99999999    .00000009    -.00000044
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[redacted]

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